DESIGN INTERACTION FORCE COEFFICIENTS OF SHEAR WALL-FRAME SYSTEMS

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DESIGN INTERACTION FORCE COEFFICIENTS OF SHEAR WALL-FRAME SYSTEMS

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CERTIFICATE

This is to certify that the work entitled "Design Interaction Force Coefficients of Shear Wall-Frame Systems" by A.Sethurathnam, has been carried out under my supervision and has not been submitted elsewhere for award of any degree.

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LIST OF SYMBOLS

CHAPTER 2

H	Total height of the structure.
M	Moment force.
P	Horizontal force.
P_{li}	Lateral load applied at i-th floor level.
Q	Vertical force.
d_c	Vector of shear wall displacements corresponding to the interaction force vector 'f_c'.
d_e	Vector of final displacements of the shear wall in any iteration cycle.
d_s	Vector of initial displacements of the shear wall in any iteration cycle.
d_t	Vector of displacements corresponding to the free deflected shape of shear wall (due to the application of total lateral loads).
f_c	Interaction force vector.
f_l	External load vector.
m	Total number of floors including ground floor
u	Lateral displacement.
v	Vertical displacement.
w	Width of shear wall.

CHAPTER 2

$\bar{w}$	Distance of the centroidal axis of the shear wall from the edge connected to the frame.
θ	Rotations.

CHAPTER 3

E	Young's modulus of elasticity.
K_s	Structure stiffness matrix.
a	Width of rectangular element.
b	Height of rectangular element.
d	Nodal displacement vector.
$\{d_r\}_i$	Displacements of the nodes on the i-th row.
f	Nodal force vector.
$\{f_r\}_i$	Applied forces on the nodes on i-th row
h	Storey height.
k	Element stiffness matrix.
M	Total number of nodes along the height of the wall.
N	Total number of nodes along the width of the wall.
$\{p\}_{ij}$	External load vector at joint 'ij'.
t	Thickness of finite element.
u	Horizontal translations.
v	Vertical translations.

CHAPTER 3

w	width of shear wall.
ϵ	Strain vector.
ν	Poisson's ratio.
σ	Stress vector.
θ	Rotations.
δE	Change in strain energy.
δW	External virtual work.

CHAPTER 4

A_b	Area of cross-section of beam.
A_c	Area of cross-section of column.
A_f	Any fixed value of area used for non-dimensionalisation.
I_b	Moment of inertia of beam.
I_c	Moment of inertia of column.
I_f	Any fixed value of moment of inertia used for non-dimensionalisation.
K_k	Structure stiffness matrix.
L_b	Beam span.
L_c	Column height.
L_f	Any fixed value of length used for non-dimensionalisation.
M	Moment force.

CHAPTER 4

P	Horizontal force.
Q	Vertical force.
$\{R_h\}_i$	Vector of external loads at the joints on the 'i'-th floor.
$R_{i,j}$	External loads at the joint i,j.
$R_{p,i}$	Vector of loads on all but one joint on the i-th floor.
$b_{i,j}$	Beam member with right hand end index i,j.
$c_{i,j}$	Column member with top end index i,j.
$\{d_h\}_i$	Vector of displacements of the joints on the i-th floor.
$d_{i,j}$	Vector of displacements of the joint, i,j.
$d_{p,i}$	Vector of displacements of all but one joint on the i-th floor.
$f_{i,j}$	Vector of forces at joint i,j.
k	Member element stiffness matrix of beam or column.
m	Number of floors in the frame (including base level).
n	Number of column lines in the frame.
$u_{i,j}$	Horizontal translation of joint i,j.
$v_{i,j}$	Vertical translation of joint i,j.

CHAPTER 4

$\left.\begin{matrix}\alpha_1, \alpha_2 \\ \alpha_3, \alpha_4\end{matrix}\right\}$	Non-dimensional parameters.
$\theta_{i,j}$	Rotation of joint i,j.

CHAPTER 5

C_{im}	Coefficients of interaction moments on frame.
C_s	Coefficients of frame shear force.
C_{wm}	Coefficients of moments on wall.
F_f	Lateral interaction forces on frame.
H	Total height of the structure.
M_b	Moment at the base of the structure due to applied loads.
M_f	Interaction moments on the frame.
M_{wi}	Moments on the wall at any level.
P	Lateral loads applied at floor levels.
V_{fx}	Shear force in the x-th storey of the frame.
V_{tx}	Total applied shear force in the x-th storey.
n_s	Total number of storeys.
α_1	Ratio of stiffness of column to beam.
α_2	Parameter representing relative stiffness of shear wall to beam.
β_1	Nondimensional parameter representing frame stiffness.
β_2	Nondimensional parameter representing relative stiffness of shear wall to the beam.

SYNOPSIS

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Provision of adequate lateral stiffness against the lateral loads due to wind, earthquake and blast effects is one of the important aspects of the design of tall buildings. The vertical reinforced concrete shear walls have been found to be one of the effective means of resisting the lateral loads. The recognition of the interaction between the walls and the frames in tall buildings results in an economical design of shear walls. The shear wall and frame system could be analysed as a planar structure, when the walls and the interconnected frames are arranged in a reasonably symmetric manner in the floor plan. In the present work, shear wall with interconnected frame is analysed as a planar structure using an iterative scheme.

In the iterative procedure, an initial deflected shape is assumed for the shear wall and the corresponding displacements of the points of connection on the shear wall with the frame

are taken as the initial values of the iteration cycle. The frame interaction forces (moments, lateral and vertical forces) required to cause the above assumed displacements of the wall on the frame are evaluated. These interaction forces are reversed in direction, applied on the shear wall and the corresponding displacements are computed. The algebraic sum of the displacements of the wall caused by the total lateral loads and the interaction forces are taken as the basis for the next iteration cycle. The displacements for the next cycle of iteration are obtained by an extrapolation technique. The iteration cycles are carried out till a required convergence of all the displacements between the initial and end values in the iteration cycle is obtained.

The solid shear wall fixed at the base behaves essentially as a cantilever beam since the height of the wall is large as compared to the width. Hence the solid shear wall is analysed by beam theory. It can be expected that the shear wall having openings upto certain sizes in each of the storeys also behaves as a cantilever. The finite element method is used to determine the limiting sizes of openings upto which the behaviour of the wall could be predicted based on beam theory. Rectangular elements with two degrees of freedom have been employed in the application of the finite element analysis to the shear wall. Walls with one row of centrally placed openings of the same size in each of the storeys are considered in the present work. It is found that the width of the openings in addition to the

overall sizes, limits the applicability of beam theory to the walls. The finite element method used for the analysis of shear walls indicates that walls with openings of sizes upto about 35% of the area of the wall could be analysed by the application of beam theory provided the width of the openings does not exceed 50% of the wall.

A non-dimensionalised form of the stiffness method of analysis has been developed to evaluate the interaction forces on the frame when it is subjected to a known set of displacements (obtained from shear wall analysis) at the points of connection with the wall. Flexural as well as axial deformations of the members of the frame are considered.

The influence of the relative stiffnesses of the wall and the members of the frame on the interaction forces between the wall and the frame has been studied. It is observed that the lateral interaction forces are very sensitive to the changes in the relative stiffnesses of the columns and beams in the frame. Curves giving the interaction forces between the shear wall (with and without openings) and the frame, in terms of non-dimensional parameters are presented for a wide range of ratios of the stiffnesses of wall to beam and column to beam. These curves would facilitate the design of frame and shear wall structures.

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CHAPTER 1

INTRODUCTION

1.1 GENERAL

With the increasing trend towards the construction of tall buildings, provision of adequate lateral stiffness against wind loads, blast effects and earthquakes has become one of the important problems in the design of these structures. The construction of reinforced concrete shear walls to act in conjunction with the building frames is one of the effective means of limiting the deflections due to lateral loads. The shear walls are interconnected to the building frames either directly or through floor diaphragms.

Two or more shear walls lying in the same plane when connected to each other by means of slabs, are named as coupled shear walls. In many buildings, the shear walls interconnected to the frames lie in parallel planes and are distributed in a reasonably symmetric manner in the floor plan. In such cases, the shear wall-frame system is analysed as a plane structure after distributing the total lateral loads in proportion to the stiffnesses of the individual shear wall-frame systems. In the early stages of design of shear wall structures, it had been assumed that the entire lateral loads were carried by the shear walls, thus neglecting the interaction between the frame and

the wall. Some investigators had replaced the frame or the wall or both by equivalent simple structures and carried out the analysis on these equivalent structures. Some others had made simplifying assumptions regarding the distribution of the interaction forces between the frame and the wall, along the height of the structure. However, due to the developments of improved methods of analysis and the evolution of high speed electronic digital computers, more rational design procedures which discard many of the simplifying assumptions made in the past, have been developed to carry out the interaction analysis of shear wall-frame systems. Quite a few publications are available for a three dimensional analysis of shear wall-frame structures considering their spatial interaction. Research has also been carried out to obtain knowledge about the stiffness contributions by the floor diaphragms connected to the shear walls, in resisting the lateral loads.

1.2 REVIEW OF LITERATURE

The solid shear wall which in general extends throughout the height of the building, behaves essentially as a cantilever beam fixed at the base. Beck (1) and Rosman (2,3) analysed coupled shear walls treating them as cantilevers interconnected by a continuous elastic medium,

instead of the discrete connecting beams or slabs at floor levels. By considering the continuous medium to be cut along mid-span and setting up compatibility of deflections at the cut sections, a second order linear differential equation was obtained in terms of the shear force in the laminae. This differential equation was solved for different boundary conditions at the base of the coupled walls. Coull et al (4,5,6) adopted the same technique and presented curves useful for the design of coupled shear walls. Many other investigations (7, 8, 9, 10, 11, 12, 13) had been carried out using the elastic continuum technique for coupled shear walls with more than one band of openings and to include the effects of variation of stiffness of wall along the height, differential settlement of foundations etc.

Some investigators (14, 15, 16, 17, 18) had treated coupled shear walls as frames and employed different methods of frame analysis like the portal method, moment distribution, influence coefficients etc. The effect of large width of the column elements viz. the walls, was accounted for by assuming the ends of the beam between the face and geometric centre line of the walls to be infinitely rigid. This idealisation is termed as 'wide column

frame analogy. In 1963, Frischmann, Prabhu and Toppler (14) used the method of influence coefficients to analyse this wide column frame, neglecting the axial and shear deformations of the members of the frame. In 1967, Jain and Chandra (15) used the flexibility method of frame analysis for coupled shear walls, considering the effects of axial and shear deformations of the walls. Gurfinkel (16) analysed coupled shear walls using a generalised cantilever moment distribution technique which automatically corrected for side-sway and required only two cycles of iteration to obtain the final results. In 1968, Jenkins and Bellamy (17) analysed coupled shear walls with an eccentric band of openings by the stiffness method of analysis. In 1970 Schwaighofer and Microys (18) suggested a modified method of wide column frame analogy in which finite values of area and second moment of area were used for the end sections of the beams embedded in the wall to represent infinite rigidity.

To obtain more reliable results regarding stress distributions, especially around openings, many authors had treated the shear wall analysis for lateral loads as a plane stress problem. Various physical discretisations like grid analogy (19), lattice model (20), finite element

technique (21,22) as well as mathematical models like line solution (23,24) finite difference solutions (25) etc. had been used for the plane stress analysis. However, this type of analysis, in general, involves large amount of computations.

A large number of publications are available on the interaction analysis of shear wall and interconnected frames. In 1960, Rosenblueth and Holtz (26) presented a numerical procedure of successive approximations in which the frame was replaced by continuous springs and the wall acted as a beam on elastic foundation. An initial deflected shape was assumed for the wall and this was improved in the successive cycles of iteration to obtain the true deflected shape. In 1961, Cardan (27) evaluated the interaction forces between the frame and the wall, assuming that the interaction forces and the lateral loads were distributed continuously throughout the height of the building. Axial deformations of the members of the frame and the wall were neglected and points of inflexion were assumed to exist at mid-span of beams and mid-height of columns. In 1964, Goyal and Sharma (28) used the stiffness method to analyse multistorey frame with an interconnected shear wall. The rigidity of the beam between the face and the

centre line of the wall was assumed to be infinite.

In 1964, Khan and Sbarounis(29) presented an iterative method for the interaction analysis of shear wall and frame systems. All shear walls were replaced by a single shear wall of equivalent stiffness and the frames were replaced by an equivalent single bay frame. The frame was rigidly connected to the shear wall by link beams at floor levels. The interaction forces considered were the moment, lateral and vertical forces. The authors also discussed the effects of torsional deformations of frames, foundation rotations, axial deformations of columns and the effective width of slab, acting along with the wall to resist lateral loads. Design curves also were presented to help in the selection of preliminary dimensions of shear wall and members of the frame. In the same year, Clough, King and Wilson (30) presented an efficient stiffness method to analyse coupled shear walls, frames and combinations of frames and shear walls subject to lateral loads. Symmetric arrangement of shear walls in the floor plan and rigid floor translations without rotations were assumed. The authors considered the shear walls as columns with finite width and their effects on girder end rotations were included.

In 1965, Gould (31) analysed combined shear wall and frame, by reducing the system to a cantilever loaded by external forces and restrained by linear and rotational springs at floor levels representing the frame. A fourth order differential equation in terms of the floor displacements was obtained as the governing equation and this was solved by finite difference technique. In 1966, Parme (32) developed an approximate method of analysis of frames interconnected with shear walls, by relating the total lateral load applied at each floor level to the displacements at that floor and the two floors above and below respectively. Shear forces in the frame and the moments on the shear walls at different levels along the height, were presented in graphical form for a wide range of structural proportions of shear wall and frame. In 1966, Tezcan (33) presented two different stiffness methods of analysis, the first an approximate and the second one relatively accurate for shear wall and frame structures. Rosman (34) replaced the shear wall by a flexural cantilever and the frame by a shear cantilever. Both these idealised cantilevers were assumed to be continuously connected by rigid laminae hinged at their ends. The analysis was carried out by setting up the complementary energy of the system and minimising it. Webster (35) presented a stiffness method of analysis of

shear walls with frames, considering the shear walls as deep columns. The author had extended the method to carry out the dynamic analysis of frames combined with shear wall.

In 1966, Thadani (36) analysed multistorey frames with shear wall, as an ordinary frame by both the flexibility and stiffness methods. The author considered the effect of shearing deformations for wall elements **but** neglected the effect of axial deformations. Thamankar, Jain and Ramasamy (37) suggested the replacement of multistorey frame inter-connected with shear walls by two cantilevers - one representing the frame and the other representing the walls. The two cantilevers were assumed to be connected at floor levels by link beams such that the sum of the shear forces at any storey in the two cantilevers was equal to the total shear acting on the building, the lateral displacements of the two cantilevers at any floor level were equal and the slopes in the ends of the link beams represented that of the cantilevers. Consideration of the above system led to a set of simultaneous equations as many as three for each floor with the end rotations of the link beams and storey shears as unknowns. In 1968, Chandra and Jain (38) presented an interaction analysis for shear wall and frame structure. In the first stage of the analysis, the lateral deflections and average storey slopes of the frames and walls were computed

by the twin cantilever method (37). In the second stage of the analysis, an iterative method was used to obtain the correct values of the slopes of different joints of the frame and thereafter the member end forces were computed. In 1970, Som and Narasimhan (39) presented a method with the assumption that the reactive forces between the frame and the wall were continuously distributed along the height. The group of shear walls and the frames were replaced by a single shear wall and an equivalent single bay frame. The interaction forces were obtained by minimising the strain energy of the frame and wall system, considering only the bending energy. Oakberg and Weaver (40) presented the analysis of rectangular frames interconnected with shear walls, using a finite element model for the latter. More details regarding this work are discussed in Chapter 3.

In 1968, Winokur and Gluck (41) presented a complete three dimensional discrete method to obtain the distribution of the lateral loads among the various stiffening elements of the multistorey structure containing shear walls, when these elements were not arranged symmetrically in the floor plan. Each floor diaphragm was assumed to be rigid in its plane. The analysis was carried out on a basic space scheme composed of vertical resisting elements and

rigid floor diaphragms. In 1970, Gluck (42) developed a three dimensional continuous method of analysis for structures consisting of prismatic or non-prismatic shear walls and frames, arranged symmetrically in plan. In this method, the frames were replaced by an equivalent continuous medium with stiffness in the main direction only, the perpendicular and torsional stiffnesses being neglected. This continuous method was compared with some discrete methods and good agreement between the results was observed.

Chriss (43) presented a three dimensional analysis for shear walls and frames in building systems based on energy principles. This method could be used for arbitrarily arranged shear walls and frames as well as for horizontal loads acting in different directions on the various faces of the wall. The strain energy equations were set up for the walls and the frames, considering torsional and translational displacements. On the application of the theorem of minimum total potential to the building system, the displacements of the different members of the system were computed as the solution of a set of linear simultaneous equations. The matrix formulation adopted in setting up the energy expressions, offered convenience to program for a computer analysis of large structures. Heidebrecht and Swift (44)

have presented a generalised three dimensional stiffness method to analyse shear wall structures. The structural system could consist of planar or non-planar shear walls distributed in an arbitrary manner and connected by beams or slabs at floor levels. The stiffness characteristics of the shear wall elements were established considering them as thin walled beams including cross-sectional warping. Axial, flexural, shear and torsional stiffnesses of the floor beams were included. Some publications (35, 45, 46) are available on the dynamic analysis of shear wall and frame systems. Research has also been carried out to understand the behaviour of the shear wall and frame systems in the inelastic range (47, 48). A comprehensive survey of the literature on different aspects of the shear wall structures is available elsewhere (49).

1.3 SCOPE OF THE PRESENT INVESTIGATION

The methods of analysis available for frame and shear wall with openings are very limited. In most of the analyses available for frame and solid shear wall systems, the wall has been modelled as a cantilever beam. The simplicity and the success experienced in the use of cantilever model for solid shear wall suggest that even shear walls with upto certain sizes of openings could be expected

to behave as cantilever walls. To determine the limiting sizes of openings upto which the application of beam theory could offer results reliable for practical purposes, a more accurate method is required. The finite element method has been used for this purpose. An iterative method is employed to carry out the interaction analysis of frame and shear wall systems. The effect of the relative stiffnesses of shear wall and frame on the interaction forces are studied. Curves offering the interaction forces between the frame and wall in terms of dimensionless coefficients for a wide range of stiffnesses of shear wall and frame systems in which shear wall has openings, are presented. The interaction forces from these curves could be used to carry out the design of the shear wall and the frame independent of each other.

CHAPTER 2

METHOD OF ANALYSIS

2.1 GENERAL

The shear wall and interconnected frame (Fig.2.1) both lying in the same plane are analysed as a planar structure, when the shear walls are distributed evenly to some extent. The shear wall which has high lateral stiffness compared to the interconnected frame, influences the deflected shape of the structure when subjected to lateral loads due to wind, earthquake etc. The analysis of the shear wall and frame system is complete if the interaction forces between the frame and the wall are computed after satisfying the equilibrium and compatibility conditions at the connection points of the frame and wall. The frames are assumed to be rigidly connected to the shear walls at floor levels, and are fixed at base. The shear wall with or without openings has the same width throughout the height of the structure but may have different thicknesses at different storeys. The forces of interaction between the frame and the wall considered in the present analysis are (i) the shear forces, (ii) the axial forces, and (iii) the moments, as indicated in Fig. 2.2. Though the

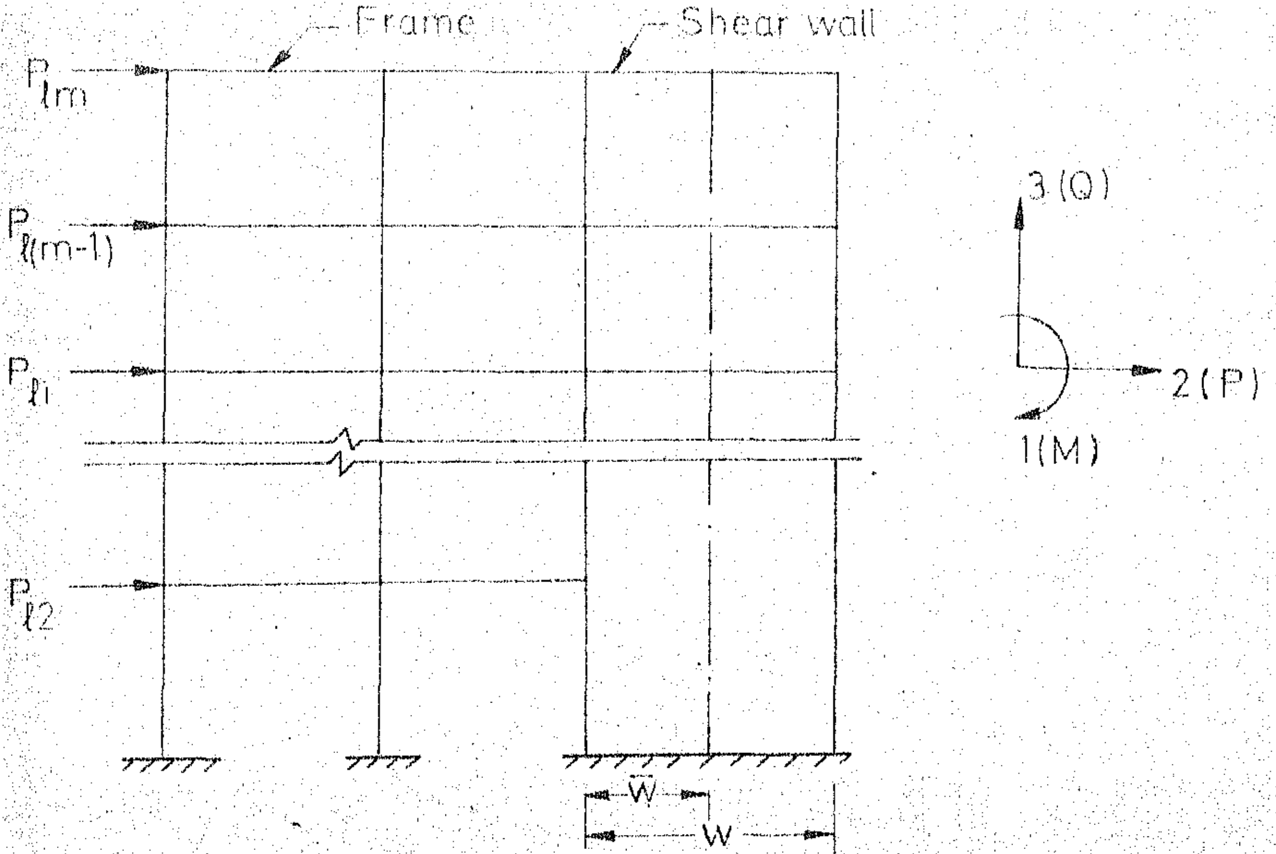
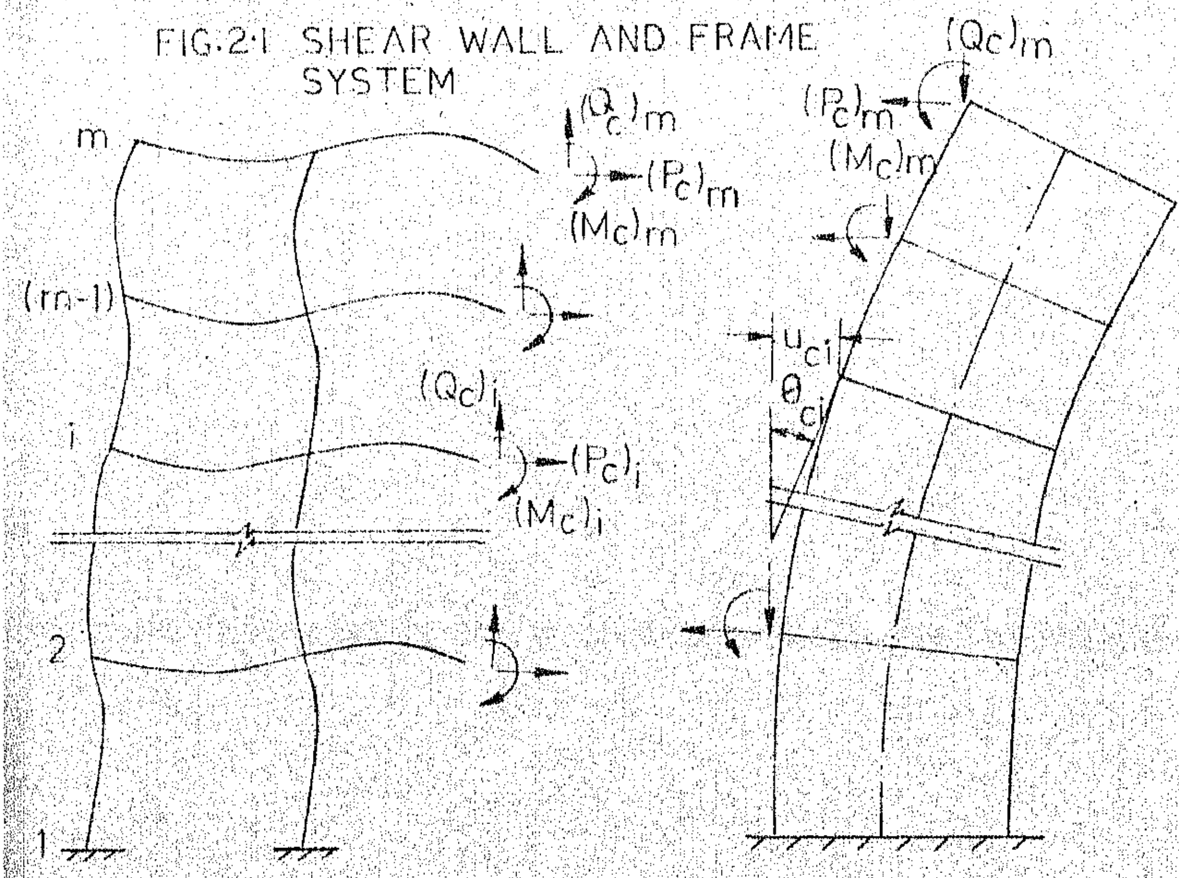


FIG. 2.1 SHEAR WALL AND FRAME SYSTEM



(a) Frame (b) Shear wall

method of analysis discussed in the next section is used primarily for lateral loads applied at floor levels, it can be used with equal convenience for the case of other types of external loads viz. vertical loads and moments, acting at the joints in the frame along with the lateral loads.

2.2 ITERATIVE SCHEME

It is convenient to consider the shear wall and frame as two separate entities and carry out the analysis to satisfy the compatibility and equilibrium conditions at the points of connection between the wall and frame. An iterative method involving simple principle is employed for the interaction analysis. The accuracy of the results obtained by this method could be improved by specifying strict convergence criteria. In this procedure, to start with, the total lateral loads f_1 acting at the floor levels, are applied on the shear wall and the corresponding displacements viz. rotations (θ), lateral translations (u) and vertical translations (v) of the wall at the points of connection with the frame are computed. The external load vector

$$f_1 = \begin{Bmatrix} f_{12} \\ f_{13} \\ \cdot \\ \cdot \\ f_{1i} \\ \cdot \\ \cdot \\ f_{1m} \end{Bmatrix} \qquad (2.1a)$$

and

$$f_{1i} = \begin{Bmatrix} 0 \\ P_{1i} \\ 0 \end{Bmatrix} \qquad (2.1b)$$

where P_{1i} refers to the lateral load at the i-th floor and 'm' is equal to the total number of floors (Fig.2.1).

The displacement vector at the connection points of the wall, due to the total lateral loads 'f_1' applied on the wall is,

$$d_t = \begin{Bmatrix} d_{t2} \\ d_{t3} \\ \vdots \\ d_{ti} \\ \vdots \\ d_{tm} \end{Bmatrix} \qquad (2.2a)$$

and

$$d_{ti} = \begin{Bmatrix} \theta_{ti} \\ u_{ti} \\ v_{ti} \end{Bmatrix} \qquad (2.2b)$$

where θ_{ti}, u_{ti} and v_{ti} refer to the rotational, lateral and vertical displacements of the wall at the i-th floor.

In the first cycle of iteration, an initial deflected shape is assumed for the shear wall with displacements of the connection points given by

$$d_s = \begin{Bmatrix} d_{s2} \\ d_{s3} \\ \cdot \\ \cdot \\ \cdot \\ d_{si} \\ \cdot \\ \cdot \\ d_{sm} \end{Bmatrix} \qquad (2.3a)$$

and

$$d_{si} = \begin{Bmatrix} \theta_{si} \\ u_{si} \\ v_{si} \end{Bmatrix} . \qquad (2.3b)$$

To maintain compatibility of displacements between the wall and the frame, the ends of the frame connected to the wall are subjected to the same set of displacements 'd_s'. The interaction forces on the frame, required to deflect it along with the shear wall are computed as

$$f_c = \begin{Bmatrix} f_{c1} \\ f_{c2} \\ \cdot \\ \cdot \\ \cdot \\ f_{ci} \\ \cdot \\ \cdot \\ \cdot \\ f_{cm} \end{Bmatrix} \qquad (2.4a)$$

and

$$f_{ci} = \begin{Bmatrix} M_{ci} \\ P_{ci} \\ Q_{ci} \end{Bmatrix} \qquad (2.4b)$$

where M_{ci}, P_{ci} and Q_{ci} refer to the moment, lateral and vertical interaction forces at the i-th floor level respectively. To enforce the conditions of equilibrium at the points of connection of the frame with the wall, the interaction force vector 'f_c' is reversed and applied on the shear wall. The displacements 'd_c' of the shear wall caused by the application of the forces '$-f_c$', are

evaluated. The algebraic sum 'd_e' of the displacements 'd_t' and 'd_c' of the shear wall gives the final set of displacements of the iteration cycle. The iteration cycles are carried out till required convergence between the initial and final sets of displacements viz. 'd_s' and 'd_e', is obtained. When a reasonable guess for the initial deflected shape of the wall could not be made, the displacement vector 'd_t' corresponding to the free deflected shape of the wall subject to the total lateral loads, could be used as initial set of displacements. The iteration scheme converges at different rates for different systems of frame and shear wall, depending on their relative stiffnesses. However, the convergence rate could be enhanced by using an extrapolation technique to obtain the initial values of the displacements for any cycle of iteration based on the displacements of the previous cycles. After the specified convergence criterion is satisfied, the interaction forces on the frame are given by the force vector 'f_c' of the last iteration cycle while the net forces on the shear wall are obtained as the algebraic sum of the force vectors corresponding to the applied external load 'f_l' and '$-f_c$' (i.e.) 'f_c' reversed in sense. With the forces on the frame and shear

wall known, the design of the frame could be carried out independent of that of the wall.

2.3 SHEAR WALL

The solid shear wall in the multistorey building has large height as compared to its width and acts essentially like a cantilever beam subject to loads at floor levels. The cantilever model for solid shear wall has been successfully employed by many investigators (29, 31). Hence, any one of the methods of beam analysis could be used to compute the deflections of solid shear walls. In the present study, the conjugate beam method (50) of analysis for the computation of deflections is employed. The shear deformations of the walls are included. While computing the deflections of the wall due to the interaction forces acting at the points of connection with the frame, the vertical edge force, 'Q_i' at any floor level 'i' (Fig. 2.3) is replaced by an axial force of the value Q_i and a moment '$Q_i \times \bar{w}$' about the centroidal axis of the wall, where $\bar{w}$ is the distance of the centroidal axis from the line of action of Q_i. The vertical displacements of the shear wall edges are computed as the sum of the axial deformations due to the vertical loads assumed

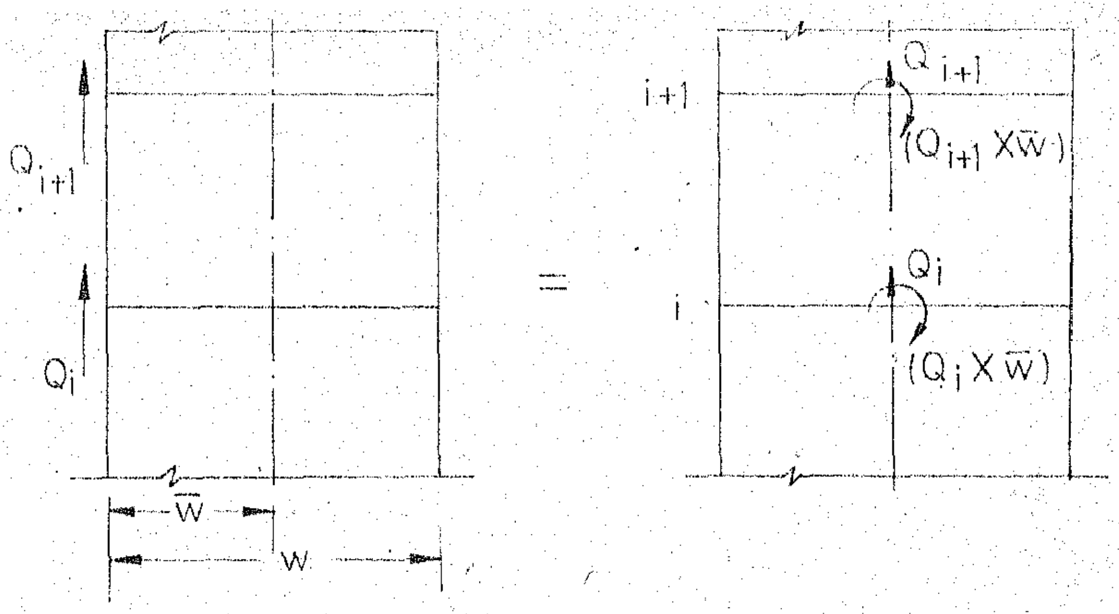


FIG. 2·3 REPRESENTATION OF VERTICAL EDGE FORCES ON SHEAR WALL BY EQUIVALENT MOMENTS AND AXIAL FORCES

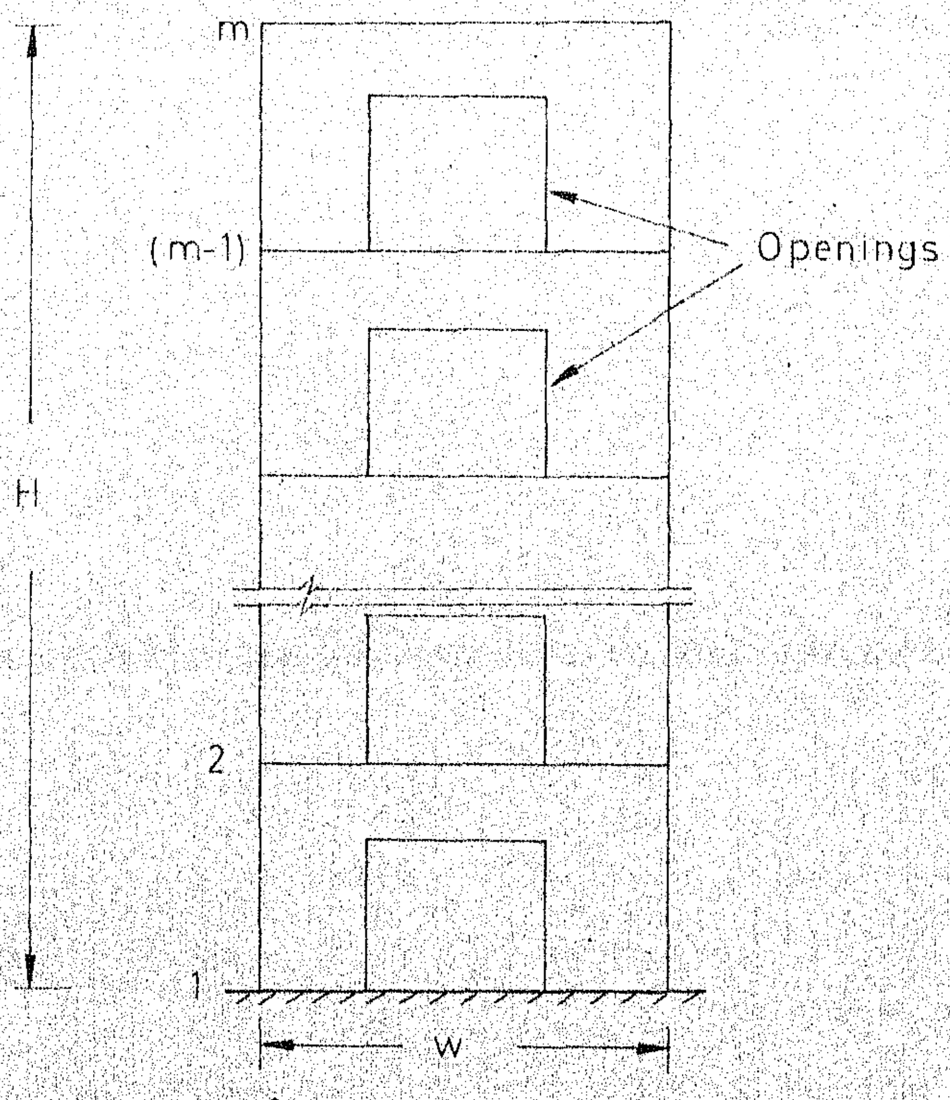


FIG. 2·4 SHEAR WALL WITH OPENINGS

to act at the centroid of the wall and the flexural displacements ($\theta \times \bar{w}$) caused by the rotations 'θ' at the respective floor levels.

Many shear walls in practice have openings to accommodate corridors, doors, windows etc. When the openings required are larger in size as in the case of corridors, the coupled shear walls consisting of two or more separate cantilever walls connected to each other through slabs or beams at floor levels are employed. But the openings required for windows and other similar purposes, are of medium size, say, in the range of 15 to 40 percent of the total area of the wall. To carry out the interaction analysis of frame and shear wall with openings, reliable methods of computing deflections of walls with openings are required. As a logical extension of the concept of analysing solid walls using the beam theory, the shear walls with openings of upto certain limiting sizes could be expected to behave as cantilevers for all practical purposes, when subject to the interaction forces. The applicability of beam theory to walls with openings of sizes exceeding the above limiting values would lead to erroneous results, as one of the main assumptions of the beam theory that the plane sections

remain plane after bending is severely violated. To determine the limiting sizes of openings, the finite element method has been employed. Walls with different sizes of openings are analysed by the conjugate beam method and the finite element method and the results of displacements are compared. The dependability and the accuracy of applying beam theory to analyse shear walls with openings are determined based on the results of the deflections computed by the finite element method. In the present investigation, walls with centrally placed single band of rectangular openings (Fig. 2.4) only are considered.

2.4 FRAMES

The method of analysis of the frame in the shear wall and frame system using the iterative technique is slightly different from the normal method of analysis of frames. That is, in the latter, the external loads acting at all the joints of the frame except at the supports are known and the displacements of these joints are required to be computed, while in the former, the displacements of the points of connection with the wall and the external loads applied at the remaining joints of the frame are known and the interaction forces at the points of connection with the wall are to be evaluated.

Hence, either the force or the displacement methods of analysis cannot be applied directly. A modified stiffness method of analysis (discussed in Chapter 4) in which the known displacements of the points of connection of the frame with the wall are eliminated from the final equations of equilibrium and a new set of equations are developed relating the known forces, and the unknown displacements of the remaining joints of the frame, has been used. This method offers convenience in programming for a computer and requires less memory space. Axial as well as flexural deformations of the members are considered both for the column and beam members of the frame. The variations in the cross-sectional properties of the members of the frame could be included without any difficulty. Any type of external joint loads viz. moment loads, vertical and horizontal loads can be handled in this analysis.

CHAPTER 3

FINITE ELEMENT METHOD - APPLIED TO SHEAR WALL

3.1 GENERAL

The finite element technique as applied to structural problems consists in the replacement of the actual structure by discrete elements with boundary forces concentrated only at the nodal points. In general, in the stiffness approach, compatibility conditions are satisfied all along the boundaries but the equilibrium is satisfied only at the nodal points. In the stiffness method, first the individual element stiffness matrices are developed for each of the elements relating the nodal forces and displacements. These element stiffness matrices of all elements are combined to obtain the total stiffness matrix of the whole structure. The nodal displacements are obtained by solving the equations of equilibrium set up at the nodal points. The finite element method is versatile one to analyse structures with complicated configurations and arbitrary boundary conditions.

In 1956, Turner et al. (51) presented stiffness matrices for triangular plate elements assuming an uniform state of strain throughout the element. They also

combined the stiffness matrices of four triangular plate elements to obtain the stiffness matrix of a quadrilateral element.

In 1960, Clough (52) analysed plates with concentrated boundary loads and plates with rectangular holes as plane stress problems using the finite element stiffness method.

In 1963, Melosh (53) discussed the development of stiffness matrices for plate elements using a displacement function and set up criteria for the selection of suitable displacement function so as to get monotonic convergence of finite element solutions.

The finite element method has been employed to analyse shear wall structures treating them as plane stress problems (21, 22, 40, 54). The application of the finite element technique to shear wall problems has been useful to include the effects of boundary columns, variations in the elastic properties of the wall along the height and to analyse the walls with openings. Girijavallabhan (21) analysed coupled shear walls using the finite element stiffness method. The author used both rectangular and triangular elements. It was observed

method of substructure employed by the authors required relatively less storage in the computer for the analysis of large structures.

In 1970, Smith, Thorburn and Tinch (55) made use of a readily available computer program for the analysis of shear wall with openings, to find the effect of stiffness changes in the piles supporting the shear wall, on the stresses in the wall. Triangular elements were used in this analysis. Dickson and Nilson (56) analysed cellular buildings consisting of vertical reinforced concrete walls and interconnected horizontal floor diaphragms, using the finite element technique. Rectangular plane stress elements with two degrees of freedom per node were employed. The inplane deformations of both the wall and floor diaphragms were considered.

3.2 ELEMENT STIFFNESS MATRIX

In the present study a rectangular finite element (Fig. 3.1) with two degrees of freedom per node, one corresponding to the vertical translations (v_i) and the other corresponding to the horizontal translations (u_i), is used. The corresponding nodal actions 'f' are shown in Fig. 3.2.

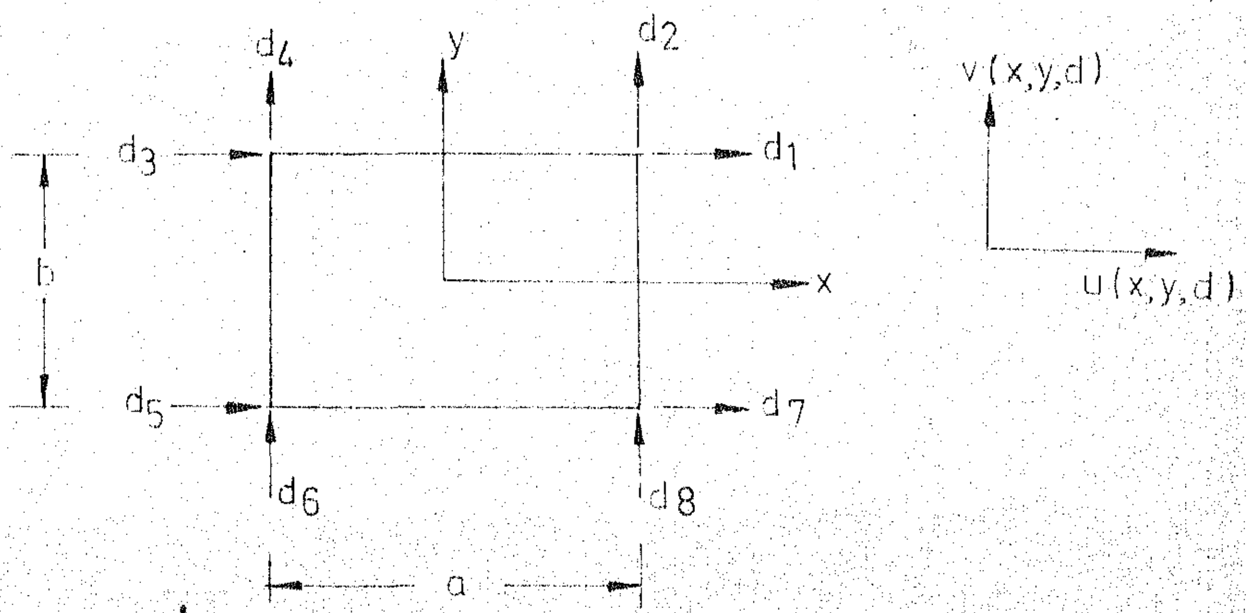


FIG. 3·1 FINITE ELEMENT

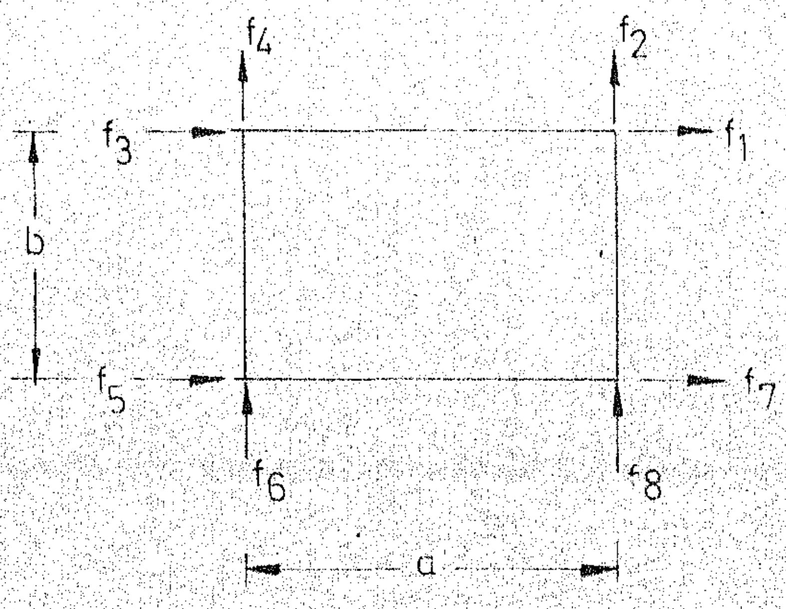


FIG. 3·2 NODAL FORCES IN THE FINITE ELEMENT

Nodal displacement vector d $= \{d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7\ d_8\}^T$

(3.1)

Nodal force vector f $= \{f_1\ f_2\ f_3\ f_4\ f_5\ f_6\ f_7\ f_8\}^T$

(3.2)

The following two displacement functions 'u' representing the horizontal translations and 'v' representing the vertical translations at any point in the element are used.

$$u = \frac{1}{ab}\left[(x - a/2)(y - b/2)\ d_1 + (x + a/2)(b/2 - y)\ d_3 + (x + a/2)(y + b/2)\ d_5 + (a/2 - x)(y + b/2)\ d_7 \right] \quad (3.3)$$

$$v = \frac{1}{ab}\left[(x - a/2)(y - b/2)\ d_2 + (x + a/2)(b/2 - y)\ d_4 + (x + a/2)(y + b/2)\ d_6 + (a/2 - x)(y + b/2)\ d_8 \right] \quad (3.4)$$

(i.e.) $\begin{Bmatrix} u \\ v \end{Bmatrix} = [\ Q\ ]\ \{d\}$ (3.5)

where

$$[\ Q\ ] = \begin{bmatrix} c_1 & 0 & c_2 & 0 & c_3 & 0 & c_4 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 & 0 & c_4 \end{bmatrix}$$

and

$$c_1 = \frac{(x - a/2)\ (y - b/2)}{ab}$$

$$c_2 = \frac{(x + a/2)\ (b/2 - y)}{ab}$$

$$c_3 = \frac{(x + a/2)\ (y + b/2)}{ab}$$

$$c_4 = \frac{(a/2 - x)\ (y + b/2)}{ab}$$

These displacement functions are linear functions of the nodal displacements and satisfy the compatibility conditions for the state of plane stress throughout the element. Further, as the displacements along any element edge are functions only of the displacements of the nodes at the ends of that edge, compatibility between adjacent elements would be maintained. The strain components for the state of plane stress are given by

$$\epsilon = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \qquad (3.6)$$

Using eq. (3.5) and rewriting eq. (3.6),

$$\epsilon = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$$= \quad R\ Q\ d \tag{3.7}$$

where

$$R = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$

The stress vector 'σ' is given by

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} \epsilon_x & \nu\epsilon_y & 0 \\ \nu\epsilon_x & \epsilon_y & 0 \\ 0 & 0 & (\frac{1-\nu}{2})\gamma_{xy} \end{bmatrix} \tag{3.8}$$

From eq. (3.7) and eq. (3.8),

$$\sigma = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$= F\ R\ Q\ d \tag{3.9}$$

where $F = \dfrac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$

The element stiffness matrix 'k' relates the nodal displacements and the corresponding nodal actions as

$$kd = f \tag{3.10}$$

The finite element with an arbitrary set of nodal displacements 'd' and corresponding nodal actions 'f' is subjected to a set of virtual nodal displacements δd. The external virtual work done by the nodal actions 'f' should be equal to the internal virtual work which is equal to the change in strain energy of the element.

The external virtual work done

$$\delta W = f^T \delta d \tag{3.11}$$

The strain components corresponding to the set of virtual nodal displacements "δd" are

$$\delta\epsilon = \begin{Bmatrix} \partial\epsilon_x \\ \partial\epsilon_y \\ \partial\gamma_{xy} \end{Bmatrix} \tag{3.12}$$

The change in the strain energy per unit volume of the element

$$= \sigma^T \delta\epsilon \tag{3.13}$$

where σ is the stress vector corresponding to the nodal displacements "d". Total change in strain energy for the whole element with uniform thickness 't',

$$\delta E = t \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \sigma^T \delta\epsilon \; dy \; dx \tag{3.14}$$

Using eqs. (3.7) and (3.9) and rewriting eq. (3.14),

$$\delta E = t \int \int d^T \; Q^T \; R^T \; F^T \; RQ\delta d \; dx \; dy \tag{3.15}$$

From eqs. (3.10) and (3.11),

$$\delta W = d^T \; k^T \; \delta d \tag{3.16}$$

But $\quad \delta W = \delta E \quad$ (3.17)

(i.e.) $\; d^T \; k^T \; \delta d = t \int \int d^T \; Q^T \; R^T \; F^T \; RQ\delta d \; dx \; dy \quad$ (3.18)

As the nodal displacements are not functions of x and y , the identity (3.18) can be written as

$$d^T \; k^T \; \delta d = d^T \; t \left[\int \int Q^T \; R^T \; F^T \; R \; Q \; dx \; dy \right] \; \delta d \tag{3.19}$$

Hence,

$$k^T = t \int \int Q^T \; R^T \; F^T \; R \; Q \; dx \; dy \tag{3.20}$$

The evaluation of the integral in eq. (3.20) gives all the coefficients of the element stiffness matrix. For the rectangular element with two degrees of freedom per node, using the displacement functions eq. (3.3) and eq. (3.4), the element stiffness matrix is evaluated from eq. (3.20). The complete element stiffness matrix for this element is shown in Table 3.1. This matrix is symmetric as required by Maxwell's theorem (57) of reciprocal deflections.

3.3 GENERATION OF STRUCTURE STIFFNESS MATRIX

The shear wall is divided into a large number of rectangular finite elements as in Fig. 3.3. The horizontal division lines are numbered starting from the bottom of the wall, while the vertical lines of division are

TABLE 3.1

ELEMENT STIFFNESS MATRIX

$$
k = \frac{Et}{4(1-\nu^2)}
\begin{bmatrix}
\frac{4(b^2+a^2r_2)}{3ab} & & & & & & & \\
(r_1 + r_2) & \frac{4(a^2+b^2r_2)}{3ab} & & & & & & \\
\frac{2(a^2r_2-2b^2)}{3ab} & (r_2 - r_1) & \frac{4(b^2+a^2r_2)}{3ab} & & & \text{(Symmetric)} & & \\
(r_1 - r_2) & \frac{(a^2-2h^2r_1)}{ab} & -(r_1 + r_2) & \frac{4(a^2+b^2r_2)}{3ab} & & & & \\
\frac{2(a^2r_2-b^2)}{3ab} & -(r_1 + r_2) & \frac{2(b^2-2a^2r_2)}{3ab} & (r_2 - r_1) & \frac{4(b^2+a^2r_2)}{3ab} & & & \\
(r_2 - r_1) & \frac{-2(a^2+b^2r_2)}{3ab} & (r_1 - r_2) & \frac{2(b^2r_2-2a^2)}{3ab} & (r_1 + r_2) & \frac{4(a^2+b^2r_2)}{3ab} & & \\
\frac{2(b^2-2a^2r_2)}{3ab} & (r_1 - r_2) & \frac{-2(b^2+a^2r_2)}{3ab} & (r_1 + r_2) & \frac{2(a^2r_2-2b^2)}{3ab} & (r_2 - r_1) & \frac{4(b^2+a^2r_2)}{3ab} & \\
(r_2 - r_1) & \frac{2(b^2r_2-2a^2)}{3ab} & (r_1 + r_2) & \frac{-2(a^2+b^2r_2)}{ab} & (r_1 - r_2) & \frac{2(a^2-2b^2r_2)}{3ab} & -(r_1 + r_2) & \frac{4(a^2+b^2r_2)}{3ab}
\end{bmatrix}
$$

where $r_1 = \nu$; $r_2 = (1 - \nu)/2.$

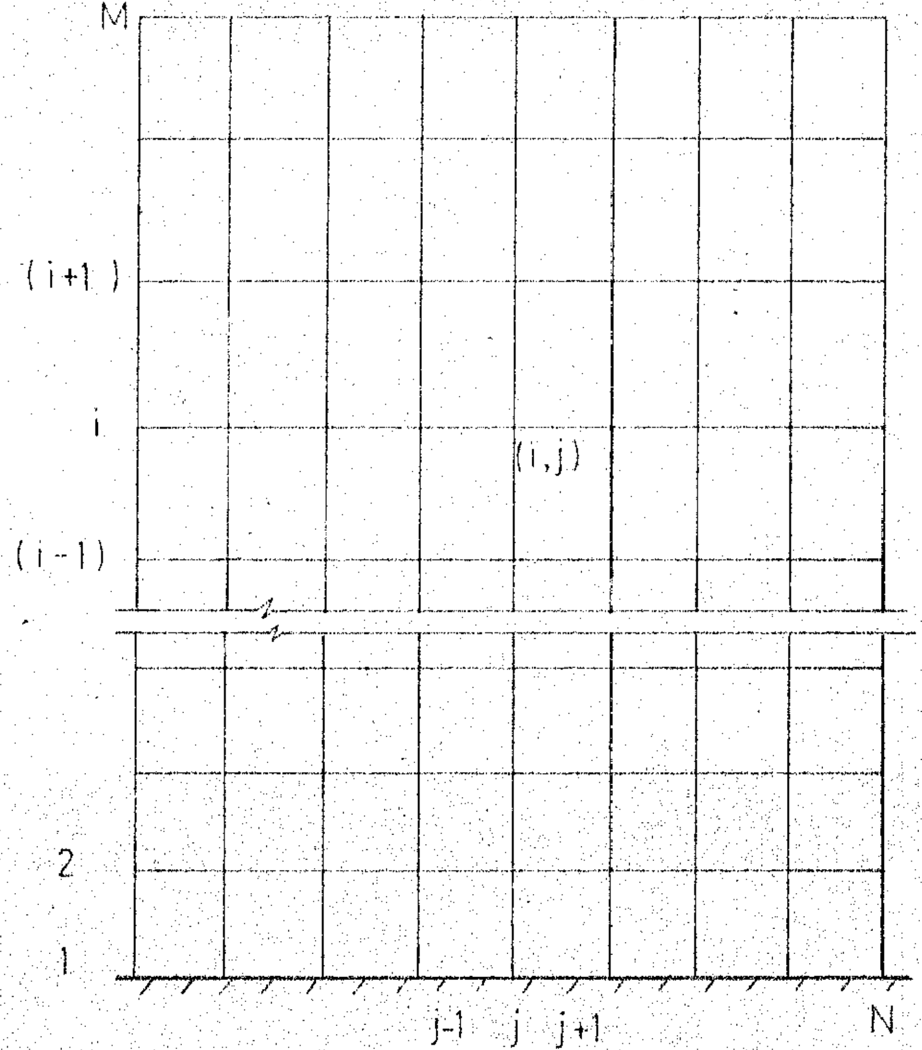


FIG.3·3 SHEAR WALL REPRESENTED BY RECTANGULAR FINITE ELEMENTS

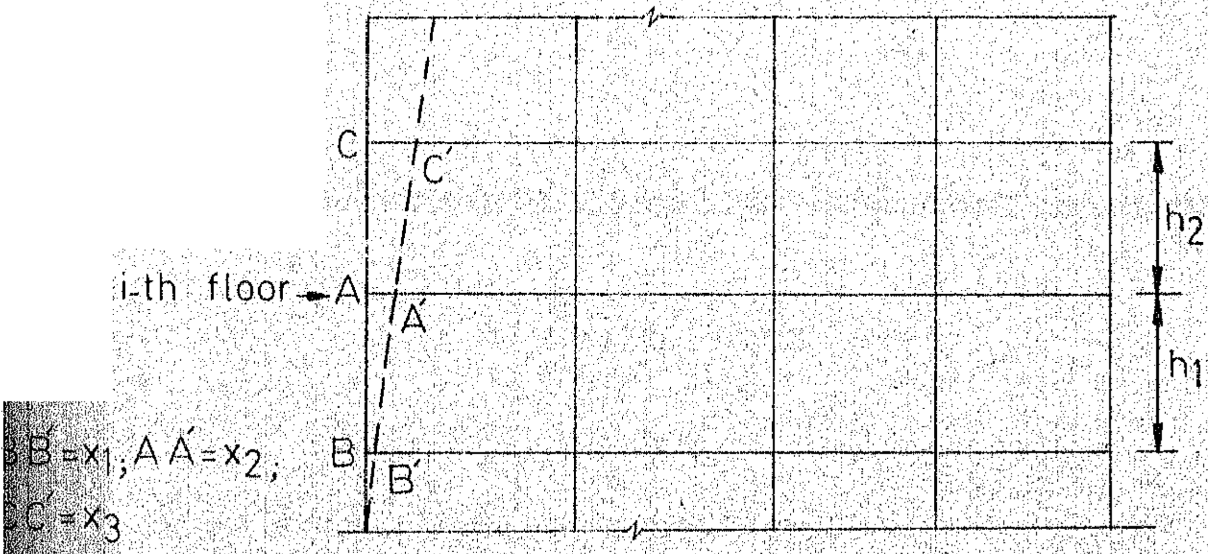


FIG. 3·4 EVALUATION OF ROTATIONS AT FLOOR LEVELS FROM LATERAL NODAL DISPLACEMENTS

numbered from left to right. It is assumed that there are M-nodes along the height of the wall and N-nodes along the width. Considering a typical node i j (Fig.3.3), the equilibrium conditions are set up at this node. The external load vector at this node is

$$\{p\}_{i,j} = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}_{i,j} \tag{3.21}$$

The sum of the nodal forces of all the four elements meeting at the node 'i j' is

$$\Sigma \{f\}_{ij} = \{f_1\}_{ij} + \{f_2\}_{ij} + \{f_3\}_{ij} + \{f_4\}_{ij} \tag{3.22}$$

For equilibrium at the node 'ij',

$$\Sigma \{f\}_{ij} = \{p\}_{ij} \tag{3.23}$$

The nodal force components in Eq. (3.22) are expressed in terms of the nodal displacements of the respective elements meeting at 'ij', using the element stiffness matrices. So Eq. (3.23) is rewritten as,

$$[\,D1 \quad D2 \quad D3\,] \begin{Bmatrix} d_{j-1} \\ d_j \\ d_{j+1} \end{Bmatrix}_{i-1} + [\,D4 \quad D5 \quad D6\,] \begin{Bmatrix} d_{j-1} \\ d_j \\ d_{j+1} \end{Bmatrix}_{i}$$

$$+ \; [\,D7 \quad D8 \quad D9\,] \begin{Bmatrix} d_{j-1} \\ d_j \\ d_{j+1} \end{Bmatrix}_{i+1} = \{p\}_{ij} \qquad (3.24)$$

where D1 to D9 are (2 x 2) sub-matrices obtained from the stiffness matrices of the elements meeting at the node 'ij', $\{d_{j-1}\}_i$, $\{d_j\}_i$ and $\{d_{j+1}\}_i$ are the displacement vectors at the nodes (i, j-1), (i,j) and (i,j+1) respectively. Equilibrium conditions generated for all the 'N' nodes lying in the i-th row, offer the following set of equations

$$[\,A\,]_i \{d_r\}_{i-1} + [\,B\,]_i \{d_r\}_i + [\,C\,]_i \{d_r\}_{i+1} = \{f_r\}_i \qquad (3.25)$$

where

$$[A]_i = \begin{bmatrix} [D2]_1 & [D3]_1 & & & \\ [D1]_2 & [D2]_2 & [D3]_2 & & \\ - & - & - & & \\ - & - & - & & \\ & [D1]_{N-1} & [D2]_{N-1} & [D3]_{N-1} \\ & & [D1]_N & [D2]_N \end{bmatrix}_i$$

$$[B]_i = \begin{bmatrix} [D5]_1 & [D6]_1 & & & \\ [D4]_2 & [D5]_2 & [D6]_2 & & \\ - & - & - & & \\ - & - & - & & \\ & [D4]_{N-1} & [D5]_{N-1} & [D6]_{N-1} \\ & & [D4]_N & [D5]_N \end{bmatrix}_i$$

$$[C]_i = \begin{bmatrix} [D8]_1 & [D9]_1 & & & \\ [D7]_2 & [D8]_2 & [D9]_2 & & \\ - & - & - & & \\ - & - & - & & \\ & [D7]_{N-1} & [D8]_{N-1} & [D9]_{N-1} \\ & & [D7]_N & [D8]_N \end{bmatrix}_i$$

$$\left\{d_r\right\}_i = \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_n \end{Bmatrix}_i$$

and

$$\left\{f_r\right\}_i = \begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ f_n \end{Bmatrix}_i$$

The matrix $[A]_i$ is obtained from the stiffness matrices of all the elements that lie just below the line 'i', $[C]_i$ is formulated out of the coefficients of the stiffness matrices of the elements just above the line 'i' while $[B]_i$ is composed from the stiffness matrices of the elements immediately above and below the line 'i'. In the development of the Eq. (3.25), it can be

observed that,

$$[C]_i = [A]^T_{i+1} \tag{3.26}$$

where the superscript 'T' stands for the transpose of the matrix. So, Eq. (3.25), representing the equations of equilibrium for all the nodes in line 'i' can be written as

$$[A]_i \{d_r\}_{i-1} + [B]_i \{d_r\}_i + [A]^T_{i+1} \{d_r\}_{i+1} = \{f_r\}_i \tag{3.27}$$

Matrices A and B are of order (2N x 2N) while d_r and f_r are of order (2N x 1).

Setting up the equilibrium conditions for all the nodes lying in each of the 'M' rows, offers the set of equations given below.

$$\begin{bmatrix} B_1 & A_2^T & & & \\ A_2 & B_2 & A_3^T & & \\ & - & - & - & \\ & - & - & - & \\ & & A_{M-1} & B_{M-1} & A_M^T \\ & & & A_M & B_M \end{bmatrix} \begin{Bmatrix} d_{r_1} \\ d_{r_2} \\ \vdots \\ \vdots \\ d_{r_{M-1}} \\ d_{r_M} \end{Bmatrix} = \begin{Bmatrix} f_{r_1} \\ f_{r_2} \\ \vdots \\ \vdots \\ f_{r_{M-1}} \\ f_{r_M} \end{Bmatrix} \tag{3.28}$$

$$(i.e.)\ K_s \quad d_s = f_s \tag{3.29}$$

where K_s , the three banded matrix is the structure stiffness matrix, d_s is the displacement vector of all nodes, and f_s is the load vector of external forces acting at all the nodes.

The stiffness matrix K_s is singular because rigid body displacements are not excluded. The inclusion of proper boundary conditions which would prevent any possible rigid body movements in the structure, makes the stiffness matrix non-singular. It is assumed that the vertical shear wall is fixed at base and hence all the displacements of the nodes in row 1 are zero. Consequently, the structure stiffness matrix K_s gets reduced to

$$\begin{bmatrix} B_2 & A_3^T & & & & \\ A_3 & B_3 & A_4^T & & & \\ & - & - & - & & \\ & - & - & - & & \\ & & A_{M-1} & B_{M-1} & A_M^T & \\ & & & A_M & B_M & \end{bmatrix} \begin{Bmatrix} d_{r_2} \\ d_{r_3} \\ , \\ , \\ , \\ , \\ d_{r_M} \end{Bmatrix} = \begin{Bmatrix} f_{r_2} \\ f_{r_3} \\ , \\ , \\ , \\ , \\ f_{r_M} \end{Bmatrix} \tag{3.30}$$

A method of forward elimination and backward substitution (58) is used to solve for the nodal displacements in Eq. (3.30). This procedure takes advantage of the banded property of the reduced stiffness matrix and avoids the storage of zero submatrices when programmed for computer solution. Further, making use of the auxiliary storage units like tapes or disks available in the computers, it is possible to avoid the simultaneous storage of all the A, B and A^T matrices except those corresponding to any one row at a time, in the core memory.

3.4 ROTATIONS AND MOMENTS

The rectangular finite elements used in the present study include only the lateral and vertical translational displacements and the nodal force components do not include moments. However, we need to compute the rotations of shear wall edge at the points of connection to the frame at floor levels to account for rotational compatibility. The rotations of the edge of the wall at these points are calculated as the average of the slopes of the vertical edges of the elements just above and below the floor levels. For example, in Fig. 3.4, the height of the element above the floor level 'i' is "h_2" while that below the floor level is h_1. x_1, x_2 and x_3 are the lateral displacements of the nodes A, B and C respectively. The slope of the wall edge with the vertical at A,

$$\theta_A = \left\{ \frac{x_3 - x_2}{h_2} + \frac{x_2 - x_1}{h_1} \right\} / 2 \tag{3.31}$$

For top corner of the wall, the rotation is calculated as the slope of the edge of the element just below the top floor level.

The interaction moment forces at floor levels on the walls are accounted by replacing the moments by two equal and opposite forces, one placed at the top node of

the element just above the floor level and the other placed at the bottom node of the element just below the floor level. For example in Fig. 3.4, a moment "M_1" at the node 'A' would be represented by a lateral force F_1 in the positive direction at 'C' and F_1 in the negative direction at 'B' such that

$$F_1 = M_1 / (h_1 + h_2) \tag{3.32}$$

The calculation of rotations from the nodal displacements and the representation of moments by an equivalent couple offer quite reliable results as would be discussed in the next section.

3.5 TESTING THE COMPUTER PROGRAM

The computer program developed to analyse vertical walls fixed at base and subjected to lateral, vertical and moment loads at each of the floor levels, using the finite element method has been checked in the following way.

The height of the wall being sufficiently larger than the width, it is expected that the deflections of the solid wall fixed at base and subjected to lateral, vertical and moment loads at floor levels can be calculated using beam theory. The results of deflections obtained using beam theory are taken as the basis to check the results of the program making use of finite element technique to compute the deflections of the same wall.

A solid shear wall of 20 storeys with a width of 200", thickness 10" and total height of 200' is loaded as shown in Fig. 3.5. The vertical load varies from 40,000 lbs at the first floor to 2,000 lbs at the top floor level, while the lateral and moment loads are the same at each of the floor levels. The lateral deflections and rotations at the points of application of the loads on the wall as obtained from the computer programs using finite element technique and conjugate beam method are

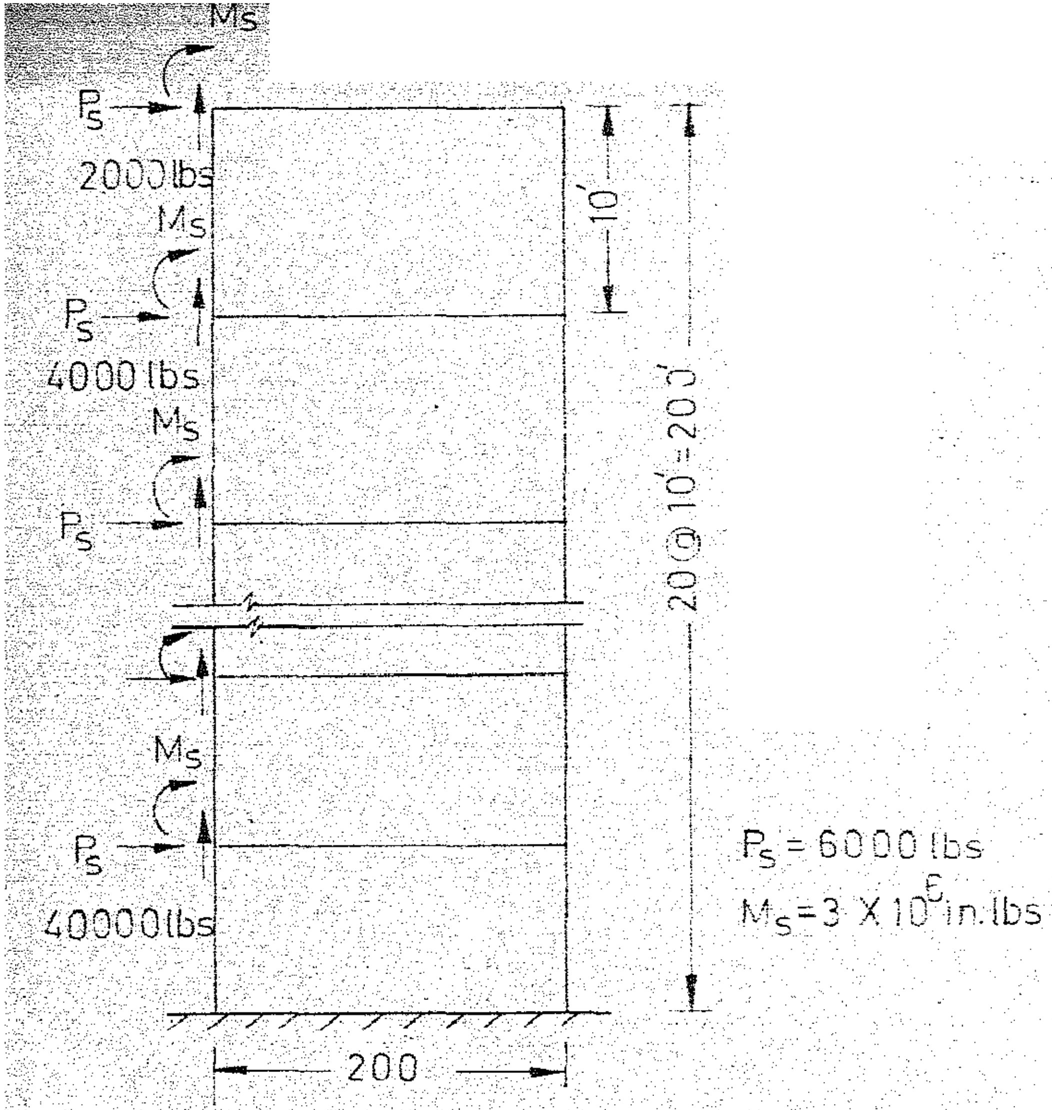


FIG. 3·5 20 STOREY SOLID WALL

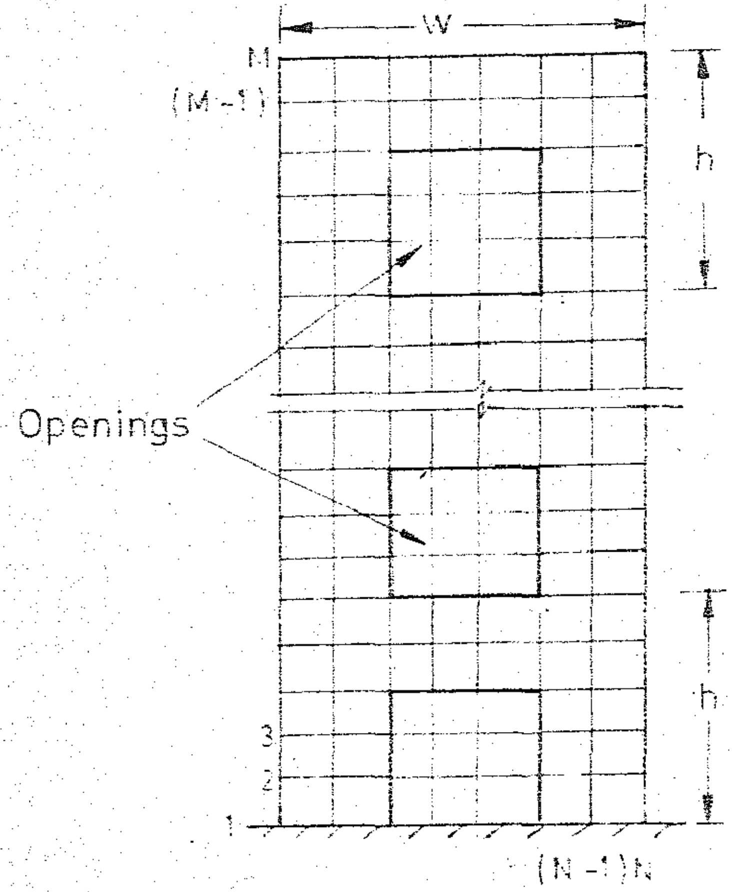


FIG. 3·6 SHEAR WALL WITH OPENINGS DIVISION INTO FINITE ELEMENTS

shown in Figs. (3.7) and (3.8). In the application of the finite element method, the shear wall was replaced by 8 elements along the width and 6 elements per storey along the height. The shear deformations were included in the calculation of the deflections of the wall using conjugate beam method. It can be observed that the lateral deflections calculated by the finite element and conjugate beam methods match very well all along the height with a maximum difference 2.16 percent. The rotations also compare very well at all floor levels except at the top floor. This is due to the fâct that in the calculation of rotations by the finite element technique, the rotation at the top floor alone is obtained as the slope of the vertical edge of the element just below the top floor and not as the average of the slopes of the elements above and below that floor level. The vertical (axial) displacements of the points of application of the loads at the floor levels compare reasonably well with a maximum deviation of 2.26 percent.

Figs. (3.7) and (3.8) also indicate the plot of lateral deflections and rotations at floor levels of a 10 storey solid wall analysed by beam theory and the finite element method. This wall is 10' wide, 8" thick

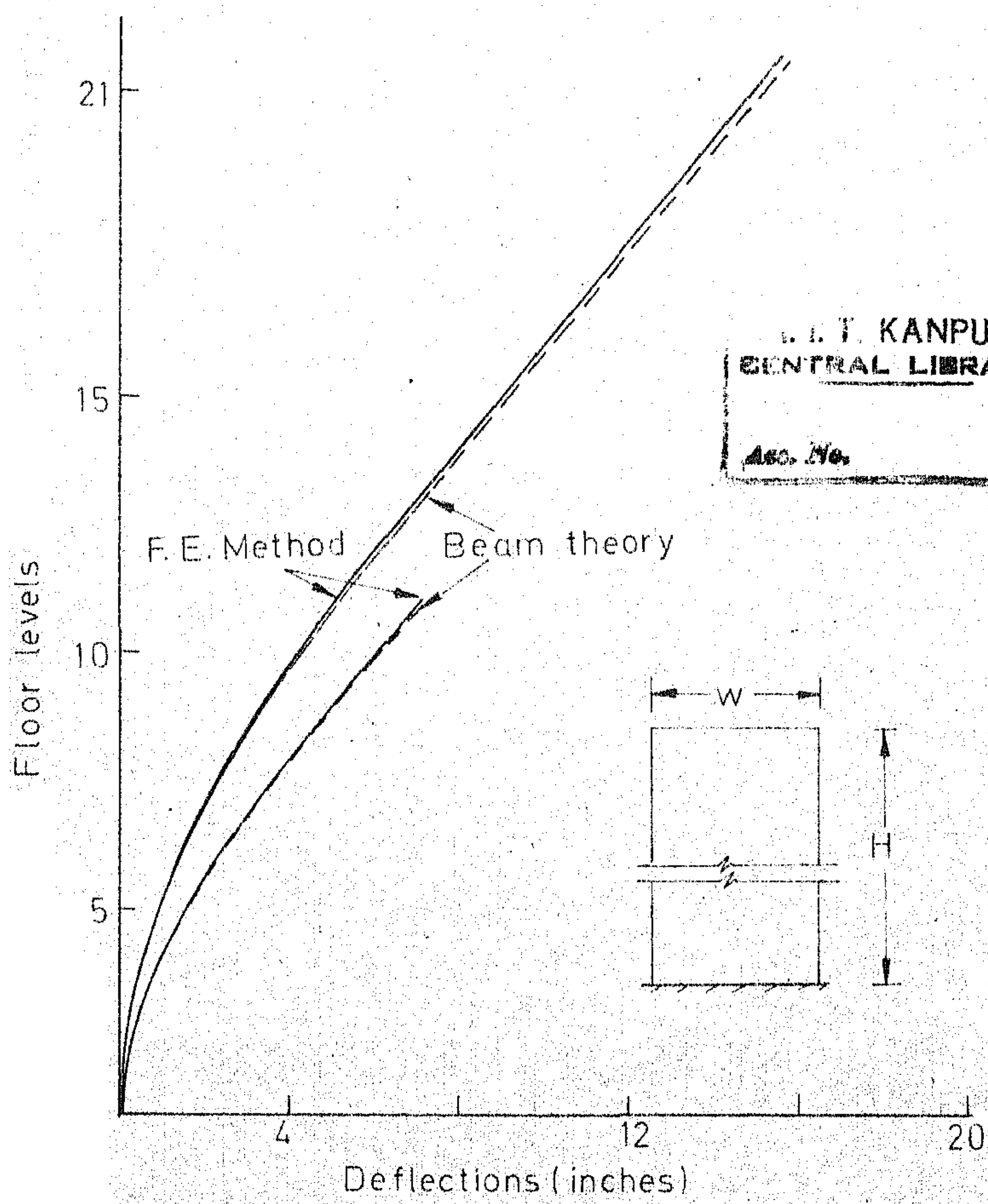


FIG. 3.7 COMPARISON OF LATERAL DEFLECTIONS OF 20 STOREY WALL BY BEAM THEORY AND FINITE ELEMENT METHOD

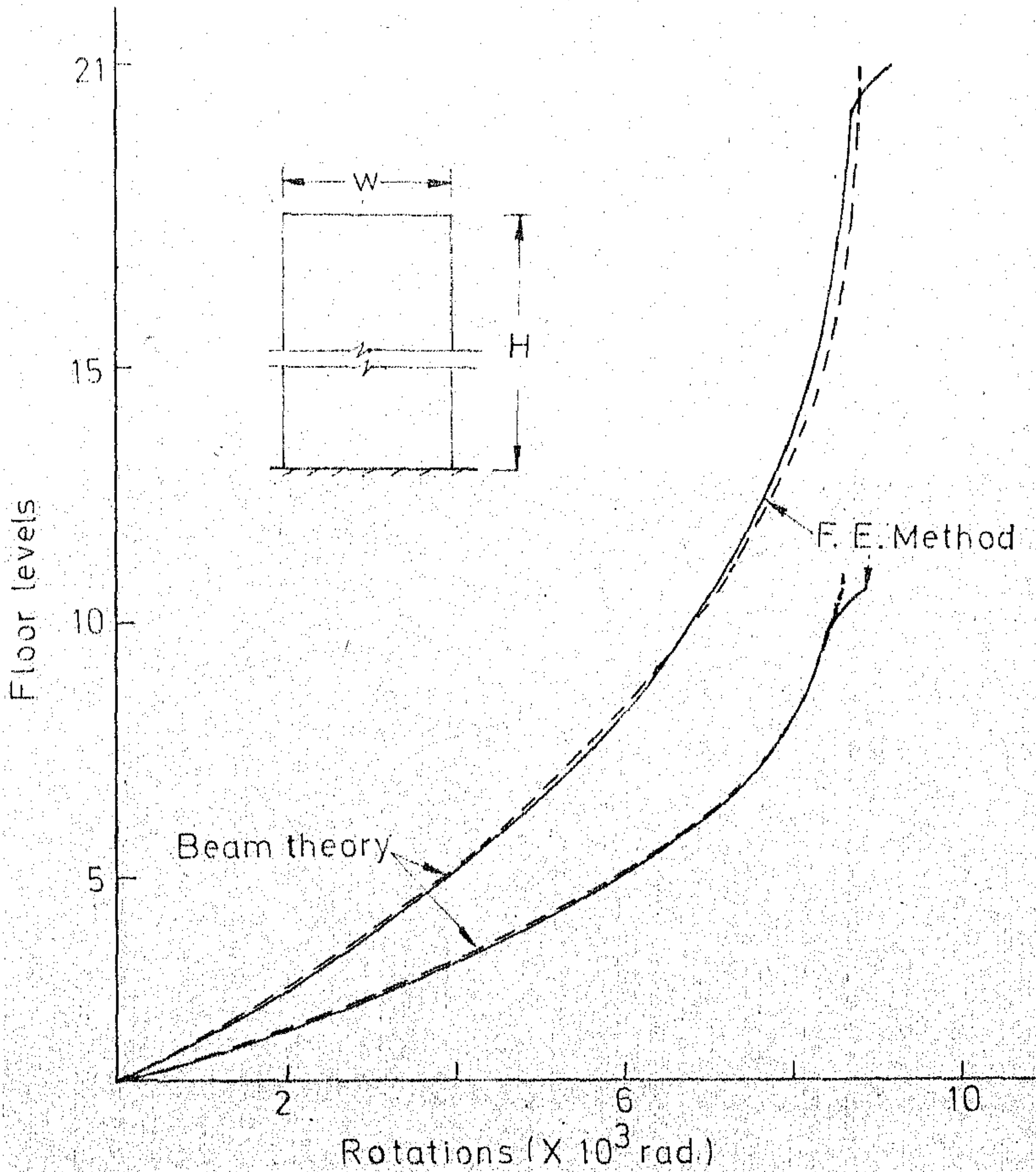


FIG.3·8 COMPARISON OF ROTATIONS OF SOLID WALL BY BEAM THEORY AND FINITE ELEMENT METHOD

and 100' high. The loads applied at the floor levels are the same as in the case of 20 storey wall except that the vertical loads varied from 20,000 lbs at the first floor level to 2000 lbs at the top floor. Very good agreement between the results of beam analysis and finite element technique can be observed in this case as well.

3.6 RELIABILITY OF BEAM THEORY FOR THE ANALYSIS OF WALLS WITH OPENINGS

The finite element program developed in the previous section can be easily modified to analyse shear walls with openings. The shear wall with openings (Fig. 3.6) is divided into rectangular finite elements such that the edges of openings coincide with the edges of the elements, that is, no part of an element in the openings extends to the adjacent solid portion of the wall surrounding the openings. These elements in the openings are assumed to have very negligible thickness (consequently negligible stiffness) as compared to the elements in the solid portion of the wall. It has been found (59) that when the elements in the openings are assumed to have about 1/40th of the thickness of the adjacent solid wall, the finite element method provides reliable results of deflections for walls with openings. If the assumed thickness for the elements in the openings is much less than about 1/40th of that of the solid wall, ill-conditioning of the stiffness matrix may develop. The modification in the computer program developed for solid walls, to account for the openings in the walls is accomplished very easily.

It has been widely accepted that the solid shear walls in the multistorey buildings when fixed at base and interconnected to the frames lying in the same plane behave essentially as a cantilever, since the height of the wall is quite large compared to its width. It is reasonable to expect similar beam type of behaviour in the case of walls with openings upto some limiting size. When the sizes of the openings in the walls exceed certain limiting values, the walls would cease to act like a beam as many of the assumptions like the plane sections remain plane before and after bending etc. are severely violated. Hence, a study is carried out to find out as to upto what limiting sizes of openings in the walls, the beam action could be assumed for the deflection analysis of the walls with openings. For this purpose, the finite element method discussed in the previous section is used. The results of the beam analysis for walls with openings are compared with those obtained by the use of finite element method to check the reliability of the former. Walls with only one band of centrally placed openings of the same size in each of the storeys are covered in this study.

20 storeyed walls (Fig. 3.9) with one band of

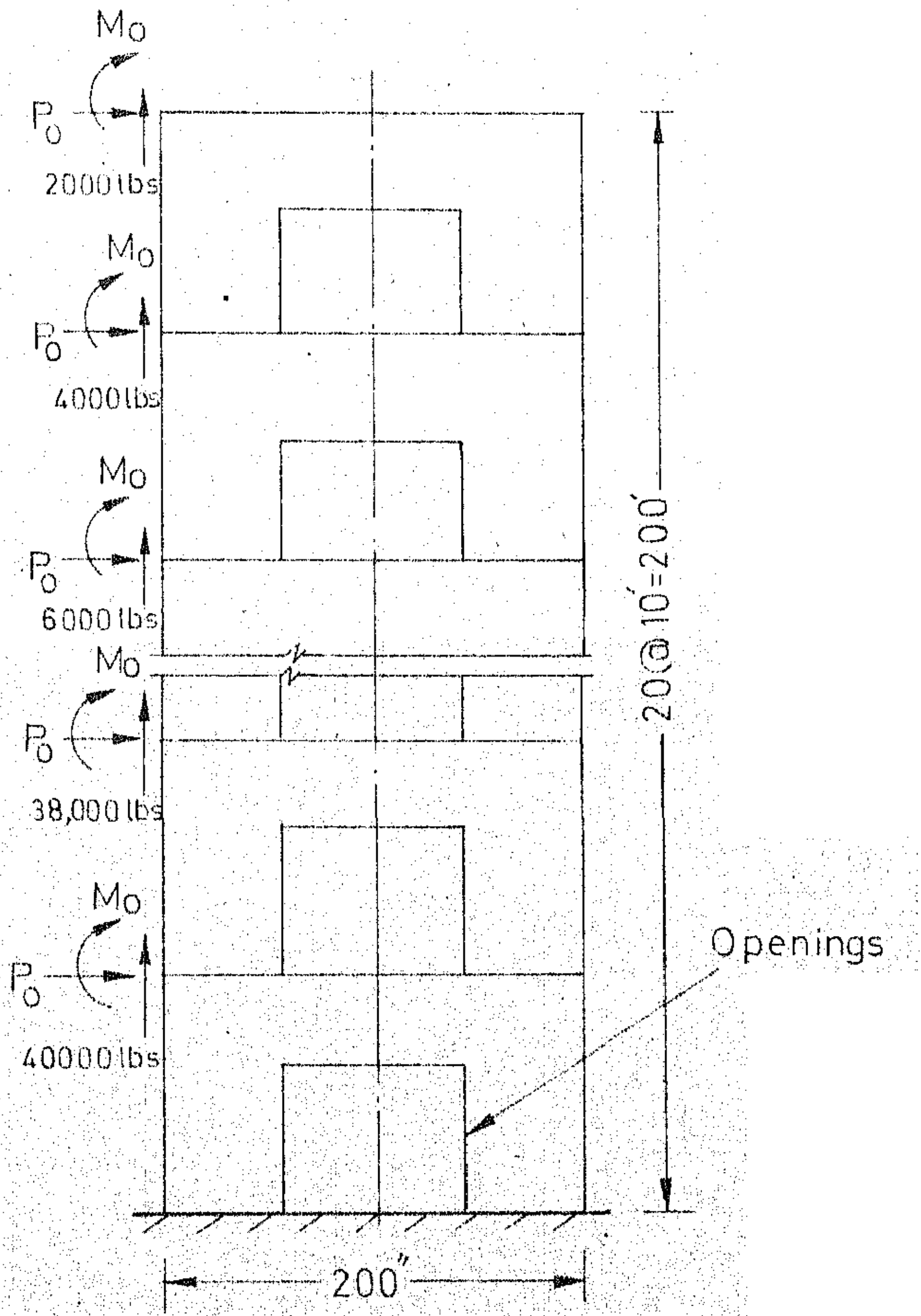


FIG. 3·9 20 STOREY SHEAR WALL WITH OPENINGS

symmetrically placed openings of same size in each of the storeys, carrying loads applied at every floor level are analysed both by the beam theory and the finite element method. The base of openings coincides with the floor levels. Different opening sizes, viz. 20 percent, 30 percent, 40 percent and 50 percent of the area of the wall are considered. Two different combinations of width and depth have been used for each of the opening sizes except the 30 percent openings size for which four different combinations of width and depth have been used. The details of the widths and depths of the openings used in this study are as follows:

(i)	20 percent openings	-	0.4w x 0.5h
			0.5w x 0.4h.
(ii)	30 percent openings	-	0.4w x 0.75h
			0.5w x 0.60h
			0.6w x 0.50h
			0.75w x 0.40h.
(iii)	40 percent openings	-	0.5w x 0.8h
			0.8w x 0.5h
(iv)	50 percent openings	-	0.625w x 0.8h.

where 'w' refers to the width of the wall and 'h' refers

to storey height. The same set of arbitrary loads used in the analysis of 20 storey solid shear wall (Fig. 3.5) is used and the lateral, vertical and rotational deflections of points of application of the loads at the floor levels have been computed both by beam theory and the finite element method. The comparison of the lateral and rotational translations of the walls with the different opening sizes are shown in Figs.(3.10) to (3.15). The plots for one set of 20 percent openings size and two sets of 30 percent openings size have not been included to avoid congestion of the curves. By studying these curves, it can be observed that the applicability of beam theory to walls with openings is restricted not only by the overall sizes of the openings but also by their width. The deviations in the deflection values of walls with 30 percent openings whose width is 0.75 times that of the wall, are appreciably greater than those in the case of 40 percent or even 50 percent openings with the width restricted to half the width of the wall. The differences between the values of the axial deflections of the walls with openings of different sizes by both the methods are not appreciable as compared to differences in the case of lateral and rotational

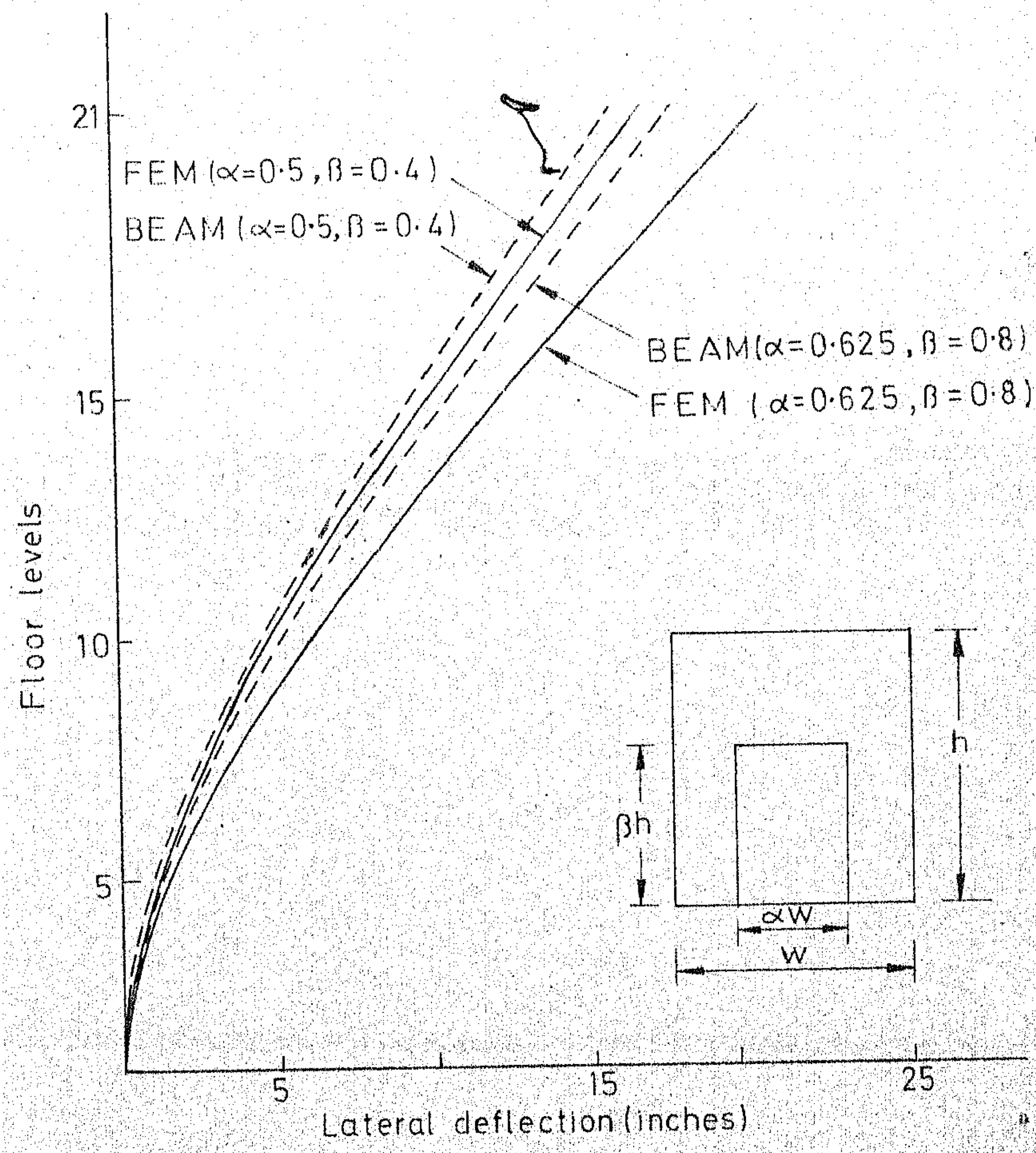


FIG. 3·10 COMPARISON OF LATERAL DEFLECTIONS OF WALLS WITH 20% & 50% OPENINGS BY BEAM THEORY AND F.E. METHOD

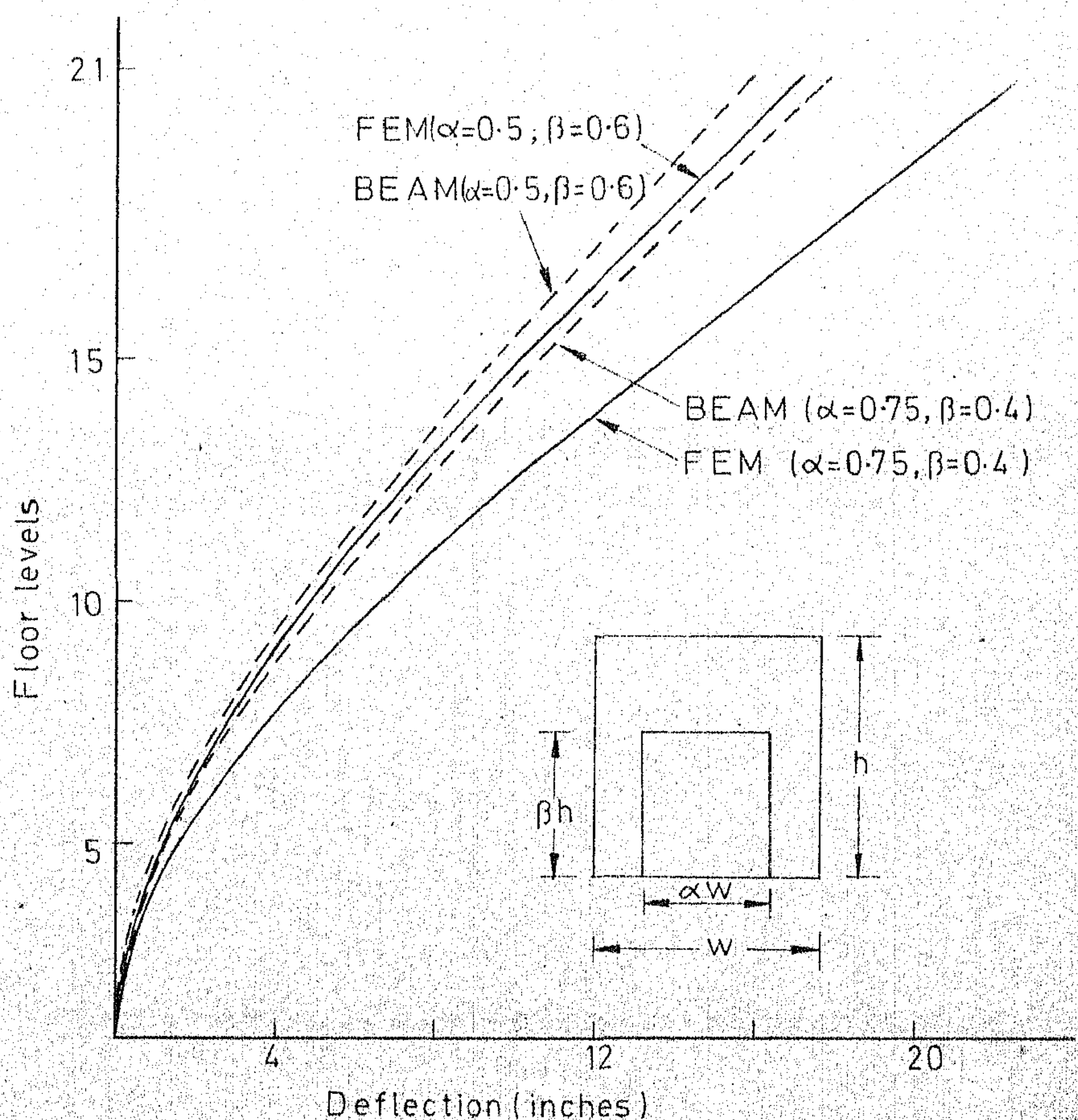


FIG. 3·11 COMPARISON OF LATERAL DEFLECTIONS OF WALLS WITH 30% OPENINGS BY BEAM THEORY AND F.E. METHOD

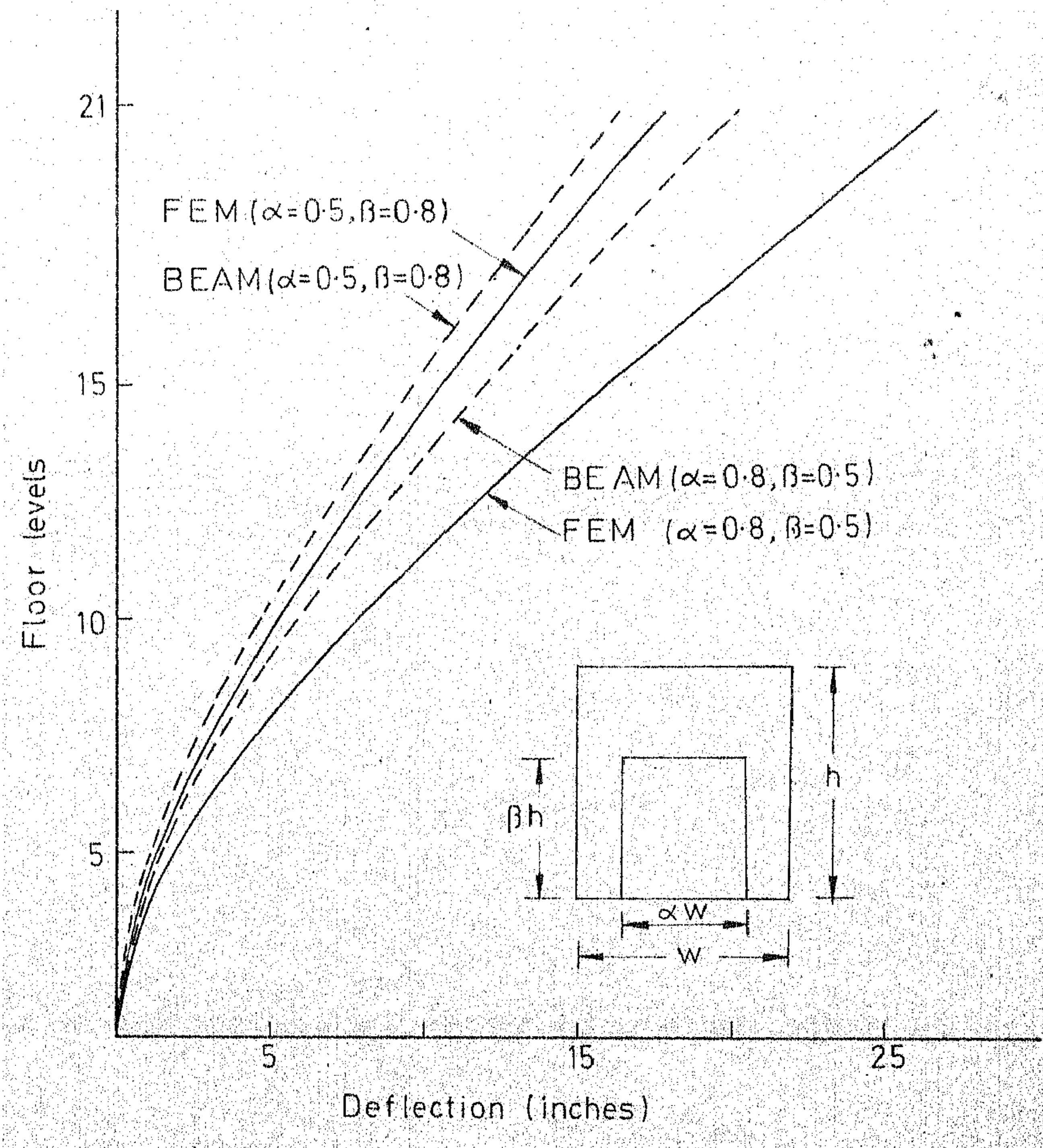


FIG 3·12 COMPARISON OF LATERAL DEFLECTIONS OF WALL WITH 40% OPENINGS BY BEAM THEORY AND FINITE ELEMENT METHOD

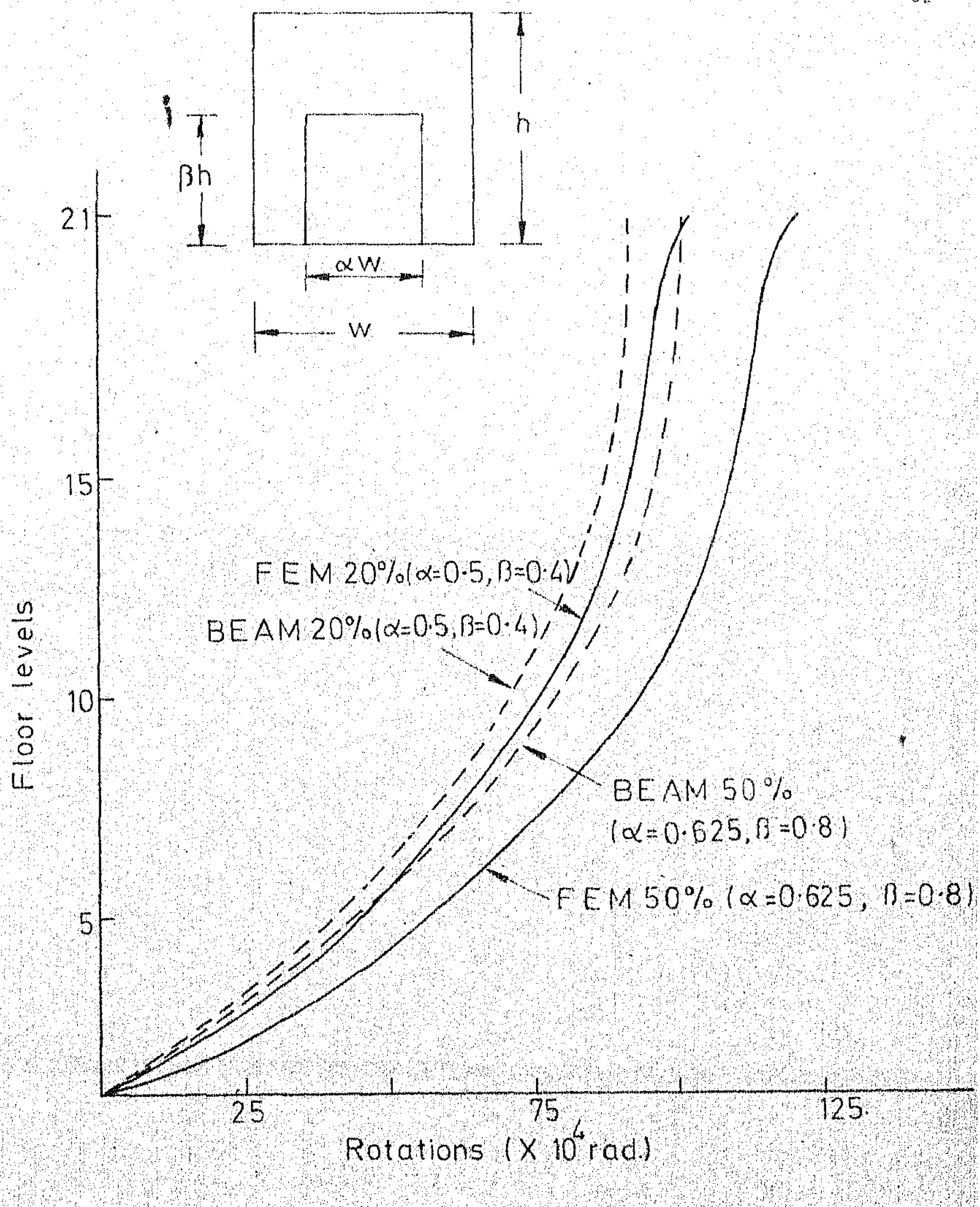


FIG. 3·13 COMPARISON OF ROTATIONS OF WALL WITH 20% AND 50% OPENINGS BY BEAM THEORY AND F.E. METHOD

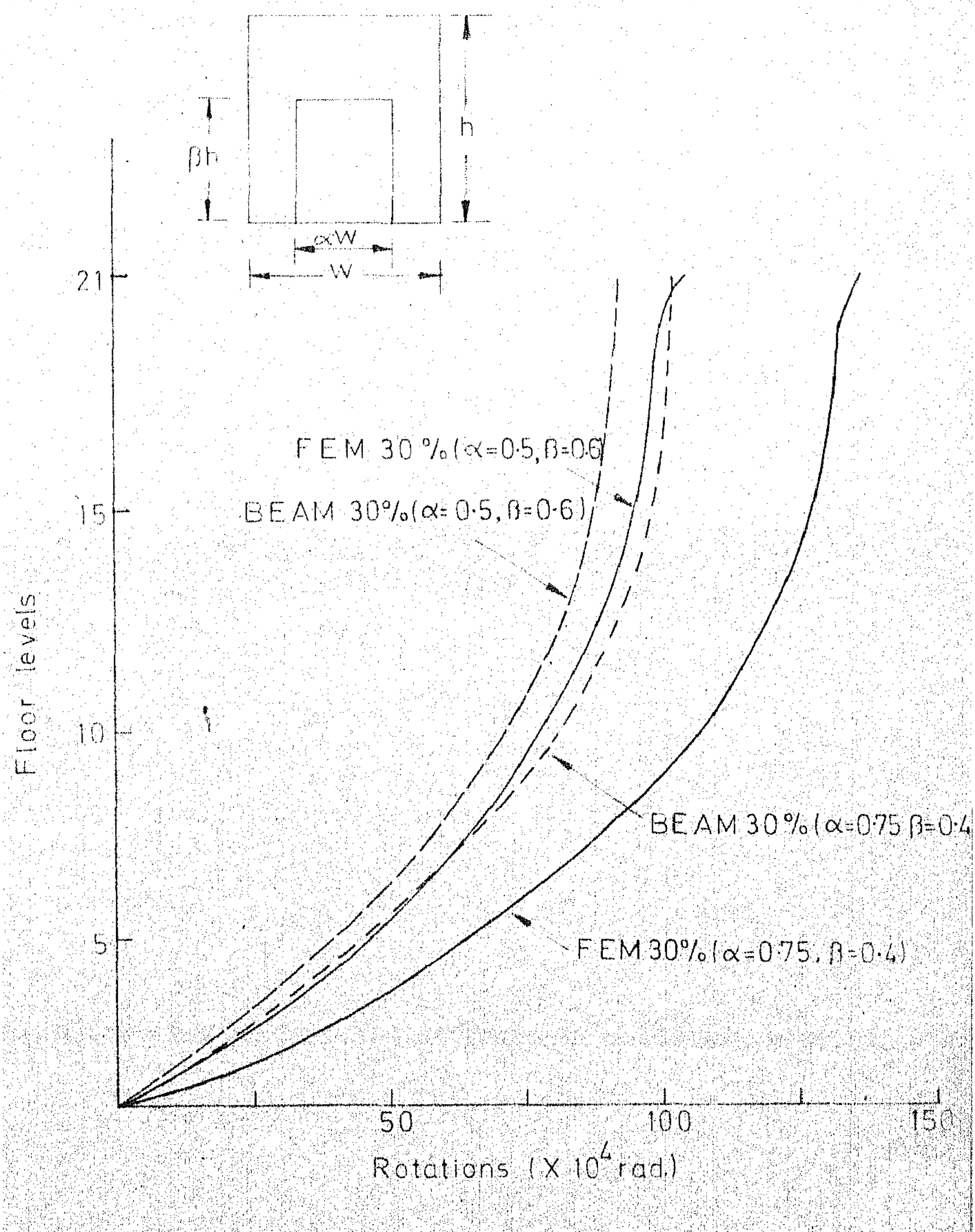


FIG.3.14 COMPARISON OF ROTATIONS OF WALLS WITH 30% OPENINGS BY BEAM THEORY AND F.E. METHOD

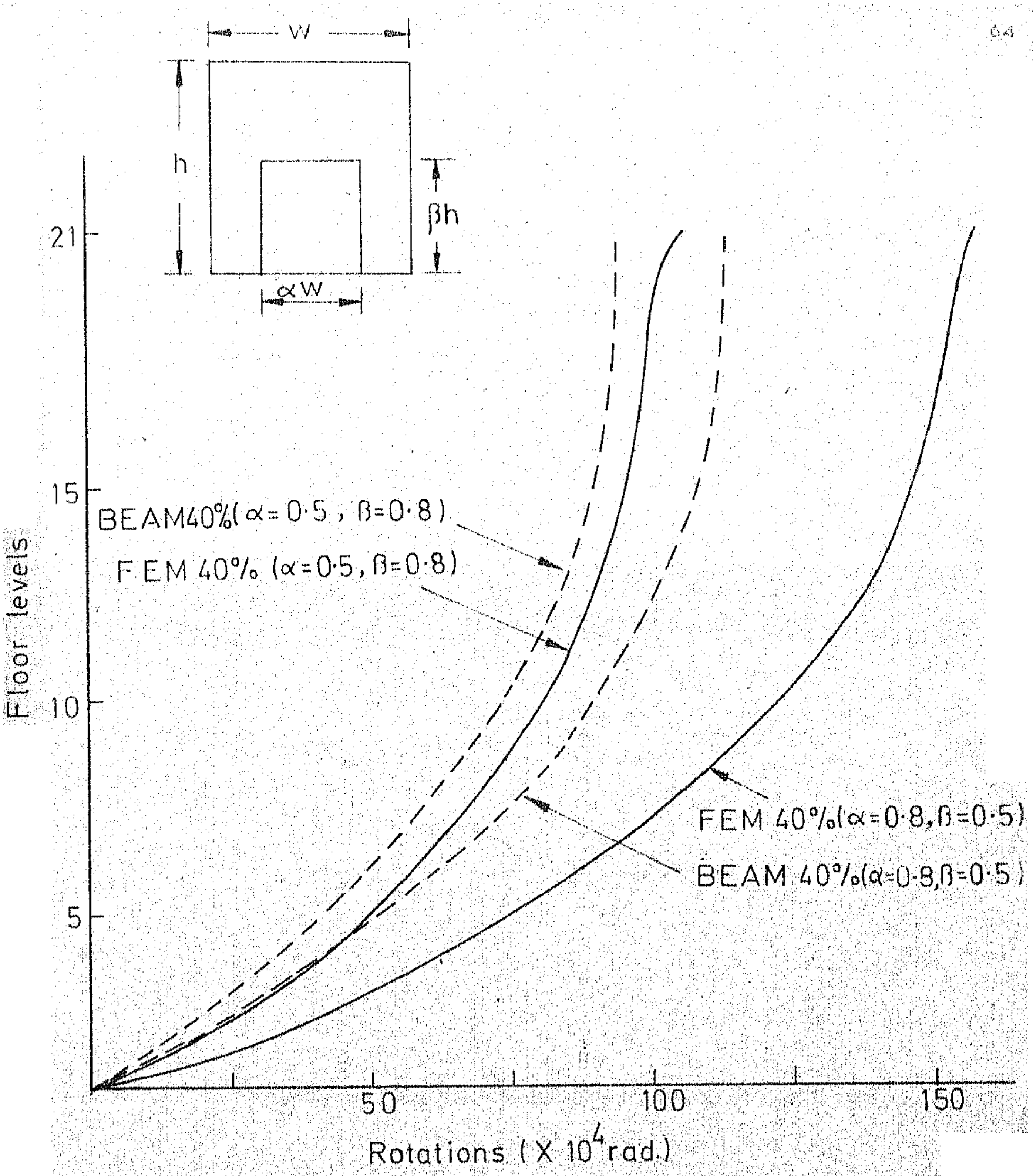


FIG. 3·15 COMPARISON OF ROTATIONS OF WALL WITH 40% OPENINGS BY BEAM THEORY AND F.E. METHOD

deformations. From the study of these graphs, it may be concluded that the beam theory is capable of offering reliable results of deflection values for all practical purposes, if the sizes of the centrally placed openings do not exceed about 35 percent of the area of wall and widths of the openings do not exceed half that of the wall.

CHAPTER 4

ANALYSIS OF FRAME

4.1 GENERAL

The recognition of the role played by the frames in carrying a part of the lateral loads applied on the shear wall-frame system leads to a realistic and economical design of these structures. The object of this chapter is to develop a non-dimensionalised form of the stiffness method of analysis, modified to suit the requirements of computing the interaction forces on the frame in the shear wall-frame system. The method is explained starting from the development of member stiffness matrices of the column and beam elements of the frame. The analysis of the shear wall also has to be carried out in terms of the non-dimensional parameters to match with the corresponding parameters involved in the analysis of frames.

ASSUMPTIONS

(i) Many buildings in practice consist of regular rectangular plane frames (Fig. 4.1) in which all the beams

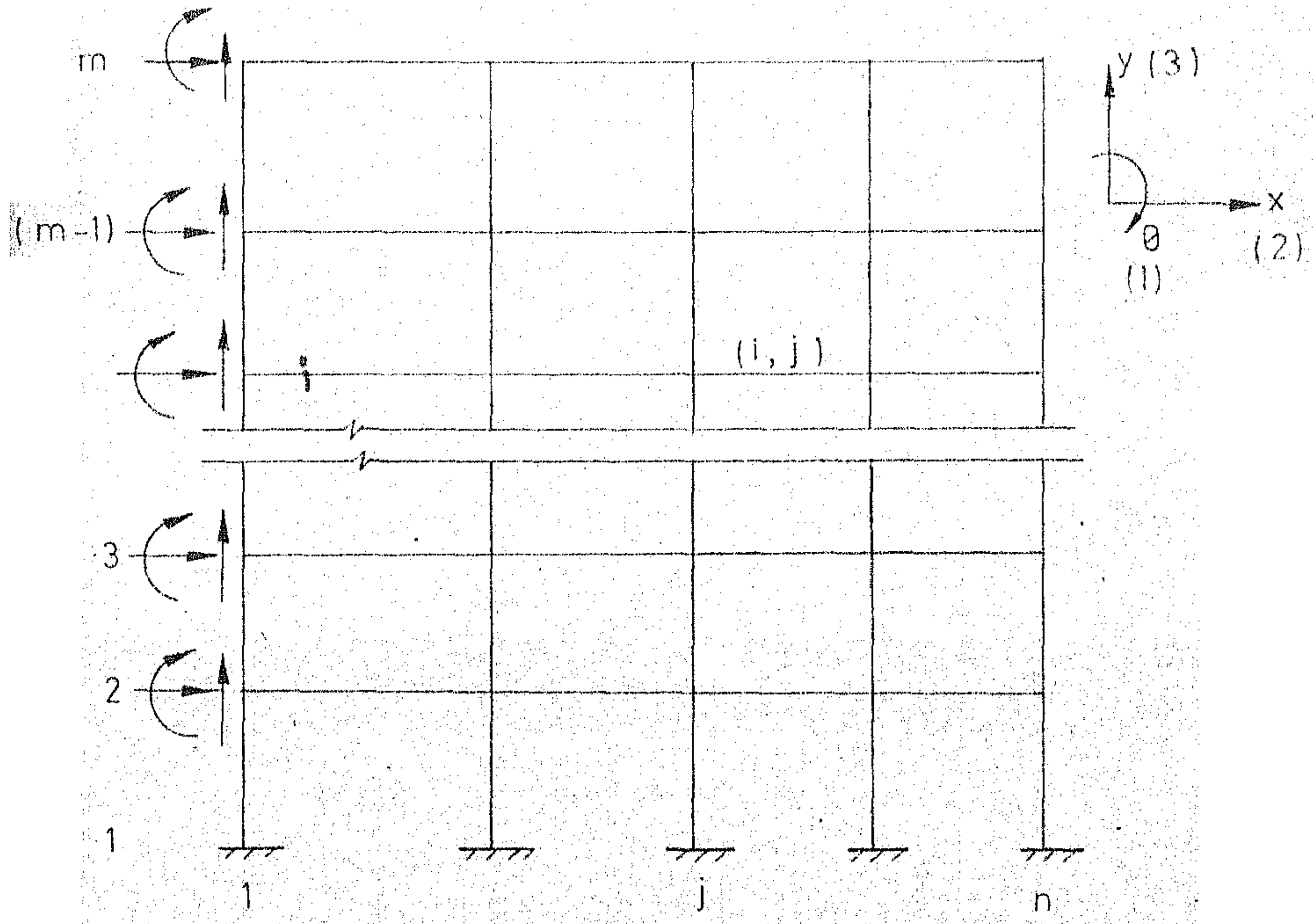


FIG. 4·1 MULTISTOREY FRAME

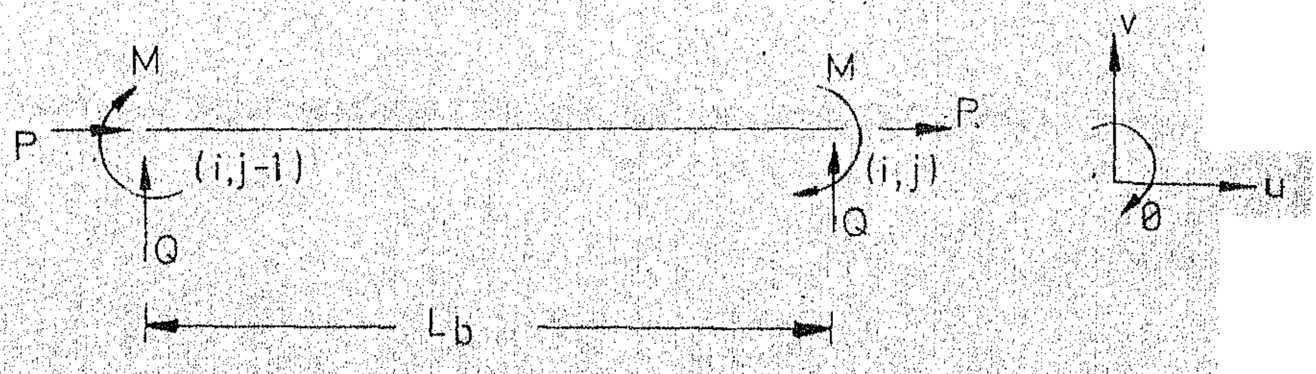


FIG. 4·2 BEAM 'b_{ij}' WITH MEMBER END FORCES

in any one bay have the same span and all columns in any particular storey possess the same height. Only such types of frames are treated in this analysis.

(ii) The loads are applied at the joints of the frame. The applied loads may include moments, horizontal and vertical loads.

(iii) The cross-sections of the beams and columns are rectangular in shape and do not vary along the length of the span. However, the variation of the section properties with length may be easily incorporated by suitably modifying the element stiffness matrices.

(iv) The behaviour of the frame is fully elastic for the range of loads considered herein.

(v) The frame is rigidly connected to the wall at floor levels and hence undergo the same displacements as those of the shear wall at the points connection.

4.2 NOTATIONS

'L' denotes the span length of beam or storey height of column members of the frame, 'A' refers to the area and 'I' refers to the second moment of area of cross-section. The suffix 'b' corresponds to beam

and 'c' represents column. For convenience in the identification of the joints of the frame, the column lines are numbered 1 to 'n' from left to right and the floor levels 1 to m from ground floor to top floor (Fig.4.1). The individual columns are designated by the index of the joint at the top of the column and the beams are identified by the index of the joint at its right hand end (Figs.4.2 and 4.3).

Initially the member stiffness matrices for the beams and columns are developed separately, then they are combined to get the joint stiffness matrices and thereafter, the structure stiffness matrix for the whole frame is developed. The structure stiffness matrix as developed above would be useful to compute the displacement vectors of the joints when the applied loads at the joints are known. However, for the interaction analysis of frame and wall system using the iteration scheme, the interaction forces on the frame when it is subjected to a known set of displacements (obtained from the analysis of shear wall) at the joints, connected to the shear wall are required. The structure stiffness matrix is modified so as to obtain the interaction forces.

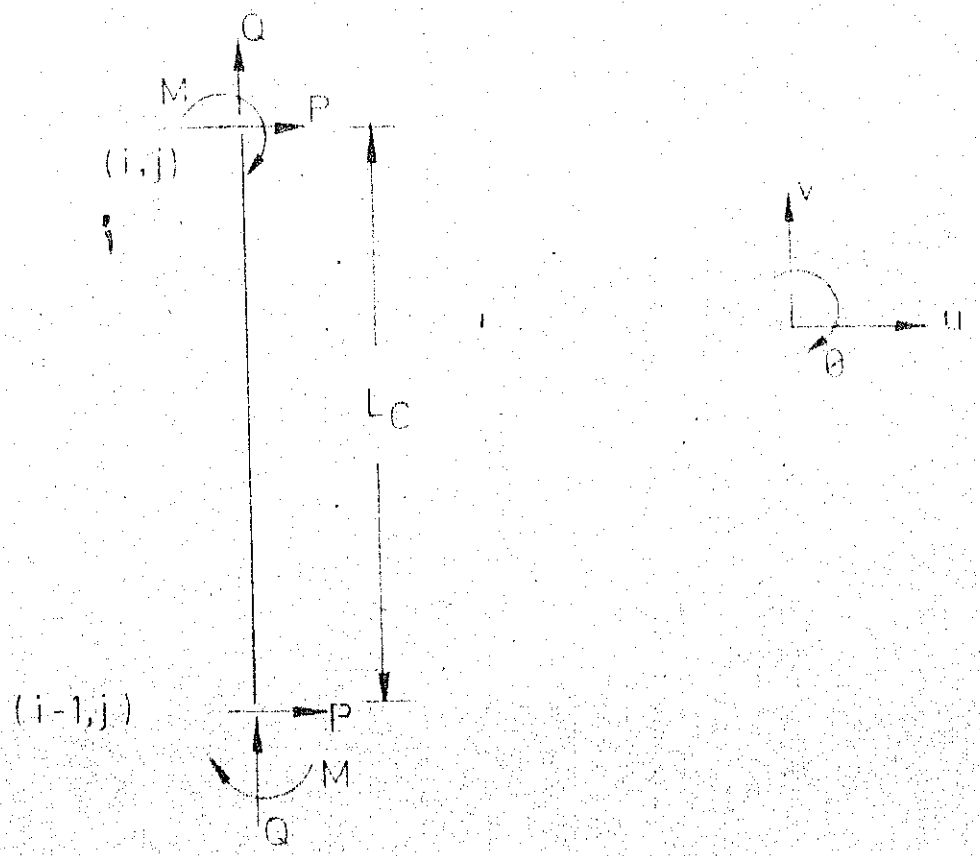


FIG.4·3 COLUMN 'C_{ij}' WITH MEMBER END FORCES

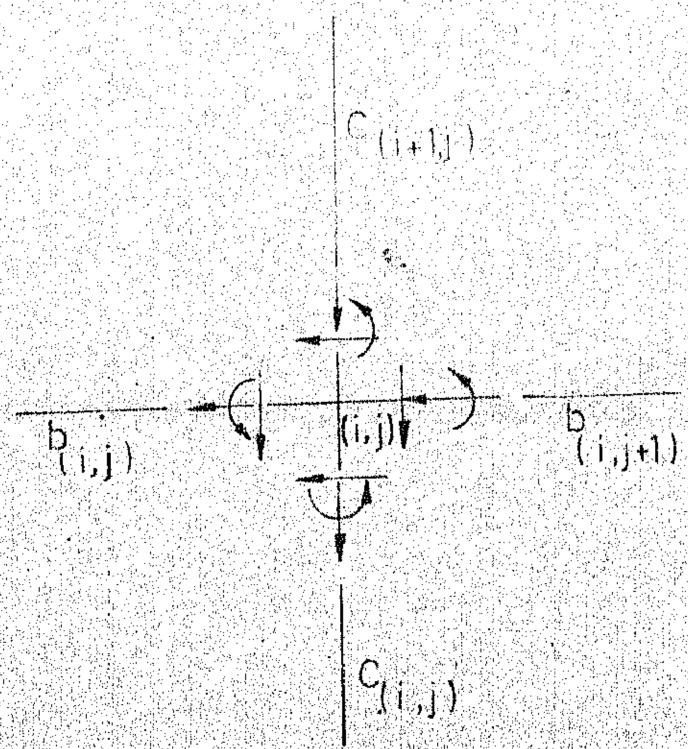


FIG. 4·4 TYPICAL FRAME JOINT (i,j)

4.3 MEMBER STIFFNESS MATRIX FOR BEAM

Considering the beam in Fig. 4.2, the forces acting at the ends are the moments 'M', axial force 'P' and the shear force 'Q'. These forces are related to the corresponding displacements 'θ', 'u' and 'v' of the ends of the beam by the following equations:

$$M_{i,j-1} = \frac{EI_b}{L_b}\left[4\,\theta_{i,j-1} + 2\theta_{i,j} - \frac{6}{L_b}(v_{i,j-1} - v_{i,j})\right] \tag{4.1a}$$

$$P_{i,j-1} = \frac{A_b E}{L_b}\left[u_{i,j-1} - u_{i,j}\right] \tag{4.1b}$$

$$Q_{i,j-1} = -\frac{6EI_b}{L_b^2}(\theta_{ij} + \theta_{i,j-1}) + \frac{12EI_b}{L_b^3}(v_{i,j-1} - v_{i,j}) \tag{4.1c}$$

$$M_{i,j} = \frac{EI_b}{L_b}\left[4\theta_{i,j} + 2\theta_{i,j-1} - \frac{6}{L_b}(v_{i,j-1} - v_{i,j})\right] \tag{4.1d}$$

$$P_{i,j} = \frac{A_b E}{L_b}(u_{i,j} - u_{i,j-1}) \tag{4.1e}$$

$$Q_{i,j} = \frac{6EI_b}{L_b^2}(\theta_{i,j} + \theta_{i,j-1}) - \frac{12EI_b}{L_b^3}(v_{i,j-1} - v_{i,j}) \tag{4.1f}$$

where E refers to the elastic modulus of the material of frame.

The above equations are rewritten in a non-dimensional form as

$$M^*_{i,j-1} = (\frac{\alpha_3}{\alpha_1}) \left[4\theta^*_{i,j-1} + 2\theta^*_{i,j} - \frac{6}{\alpha_1} (v^*_{i,j-1} - v^*_{i,j}) \right] \tag{4.2a}$$

$$P^*_{i,j-1} = \frac{\alpha_4\alpha_2}{\alpha_1} \left[u^*_{i,j-1} - u^*_{i,j} \right] \tag{4.2b}$$

$$Q^*_{i,j-1} = -\frac{6\alpha_3}{\alpha_1^2} (\theta^*_{i,j} + \theta^*_{i,j-1}) + \frac{12\alpha_3}{\alpha_1^3} (v^*_{i,j-1} - v^*_{i,j}) \tag{4.2c}$$

$$M^*_{i,j} = (\frac{\alpha_3}{\alpha_1}) \left[2\theta^*_{i,j-1} + 4\theta^*_{i,j} - \frac{6}{\alpha_1} (v^*_{i,j-1} - v^*_{i,j}) \right] \tag{4.2d}$$

$$P^*_{i,j} = \frac{\alpha_4\alpha_2}{\alpha_1} \left[u^*_{i,j} - u^*_{i,j-1} \right] \tag{4.2e}$$

$$Q^*_{i,j} = \frac{6\alpha_3}{\alpha_1^2} (\theta^*_{i,j} + \theta^*_{i,j-1}) - \frac{12\alpha_3}{\alpha_1^3} (v^*_{i,j-1} - v^*_{i,j}) \tag{4.2f}$$

$$M^*_{ij} = \frac{M_{i,j}\, L_f}{E\, I_f} ;$$

$$P^*_{ij} = \frac{P_{i,j}\, L_f^2}{E\, I_f} ;$$

$$Q^*_{i,j} = \frac{Q_{i,j}\, L_f^2}{E\, I_f} ;$$

$$\theta^*_{i,j} = \theta_{i,j} ;$$

$$u^*_{i,j} = \frac{u_{i,j}}{L_f} ;$$

$$v^*_{i,j} = \frac{v_{i,j}}{L_f} ;$$

$$\alpha_1 = \frac{L_b}{L_f} ;$$

$$\alpha_2 = \frac{A_b}{A_f} ;$$

$$\alpha_3 = \frac{I_b}{I_f} ;$$

$$\alpha_4 = \frac{A_f\, L_f^2}{I_f} ;$$

L_f, being any fixed length, A_f a fixed value of area and I_f second moment of area of cross-section, adopted for non-dimensionalisation.

Rewritten in matrix notations, Eqs. (4.2) become,

$$[\,k\,]_{bij} \quad \{d\}_{bij} \quad = \quad \{f\}_{bij} \tag{4.3}$$

$$[\,k\,]_{bij} = \begin{bmatrix} \frac{4\alpha_3}{\alpha_1} & 0 & \frac{6\alpha_3}{\alpha_1^2} & \frac{2\alpha_3}{\alpha_1} & 0 & \frac{6\alpha_3}{\alpha_1^2} \\ 0 & \frac{\alpha_4\alpha_2}{\alpha_1} & 0 & 0 & -\frac{\alpha_4\alpha_2}{\alpha_1} & 0 \\ -\frac{6\alpha_3}{\alpha_1^2} & 0 & \frac{12\alpha_3}{\alpha_1^3} & -\frac{6\alpha_3}{\alpha_1^2} & 0 & -\frac{12\alpha_3}{\alpha_1^3} \\ \frac{2\alpha_3}{\alpha_1} & 0 & -\frac{6\alpha_3}{\alpha_1^2} & \frac{4\alpha_3}{\alpha_1} & 0 & \frac{6\alpha_3}{\alpha_1^2} \\ 0 & -\frac{\alpha_4\alpha_2}{\alpha_1} & 0 & 0 & \frac{\alpha_4\alpha_2}{\alpha_1} & 0 \\ \frac{6\alpha_3}{\alpha_1^2} & 0 & -\frac{12\alpha_3}{\alpha_1^3} & \frac{6\alpha_3}{\alpha_1^2} & 0 & \frac{12\alpha_3}{\alpha_1^3} \end{bmatrix}$$

$$\{d\}_{bij} = \begin{Bmatrix} d_{i,j-1} \\ d_{i,j} \end{Bmatrix} = \begin{Bmatrix} \theta^*_{i,j-1} \\ u^*_{i,j-1} \\ v^*_{i,j-1} \\ \theta^*_{i,j} \\ u^*_{i,j} \\ v^*_{i,j} \end{Bmatrix}$$

$$\{f\}_{bij} = \begin{Bmatrix} M^*_{i,j-1} \\ P^*_{i,j-1} \\ Q^*_{i,j-1} \\ M^*_{i,j} \\ P^*_{i,j} \\ Q^*_{i,j} \end{Bmatrix}$$

$[k]_{bij}$ is the member stiffness matrix of the beam bij and $\{d\}_{bij}$ and $\{f\}_{bij}$ are the displacement and force vectors at the ends of the beam respectively.

4.4 MEMBER STIFFNESS MATRIX FOR THE COLUMN

The displacements and forces at the ends of the column, c_{ij} (Fig. 4.3) are related as

$$[\,k\,]_{c_{ij}} \{d\}_{c_{ij}} = \{f\}_{c_{ij}} \tag{4.4}$$

where the member stiffness matrix for the column

$$[K]_{cij} = \begin{bmatrix} \frac{4\alpha_3}{\alpha_1} & \frac{6\alpha_3}{\alpha_1^2} & 0 & \frac{2\alpha_3}{\alpha_1} & -\frac{6\alpha_3}{\alpha_1^2} & 0 \\ \frac{6\alpha_3}{\alpha_1^2} & \frac{12\alpha_3}{\alpha_1^3} & 0 & \frac{6\alpha_3}{\alpha_1^2} & -\frac{12\alpha_3}{\alpha_1^3} & 0 \\ 0 & 0 & \frac{\alpha_4\alpha_2}{\alpha_1} & 0 & 0 & -\frac{\alpha_4\alpha_2}{\alpha_1} \\ \frac{2\alpha_3}{\alpha_1} & \frac{6\alpha_3}{\alpha_1^2} & 0 & \frac{4\alpha_3}{\alpha_1} & -\frac{6\alpha_3}{\alpha_1^2} & 0 \\ -\frac{6\alpha_3}{\alpha_1^2} & -\frac{12\alpha_3}{\alpha_1^3} & 0 & -\frac{6\alpha_3}{\alpha_1^2} & \frac{12\alpha_3}{\alpha_1^3} & 0 \\ 0 & 0 & -\frac{\alpha_4\alpha_2}{\alpha_1} & 0 & 0 & \frac{\alpha_4\alpha_2}{\alpha_1} \end{bmatrix}$$

the displacements vector

$$\{d\}_{c_{ij}} = \begin{Bmatrix} \theta^*_{i-1,j} \\ u^*_{i-1,j} \\ v^*_{i-1,j} \\ \theta^*_{i,j} \\ u^*_{i,j} \\ v^*_{i,j} \end{Bmatrix}$$

and the force vector

$$\{f\}_{c_{ij}} = \begin{Bmatrix} M^*_{i-1,j} \\ P^*_{i-1,j} \\ Q^*_{i-1,j} \\ M^*_{i,j} \\ P^*_{i,j} \\ Q^*_{i,j} \end{Bmatrix}$$

As before M^*, P^*, Q^* represent the nondimensionalised form of forces, θ^*, u^*, v^* represent the non-dimensionalised displacements. Also, $\alpha_1 = \frac{L_c}{L_f}$; $\alpha_2 = \frac{A_c}{A_f}$; $\alpha_3 = \frac{I_c}{I_f}$ and

$$\alpha_4 = \frac{A_f L_f^2}{I_f} ,$$

L_f, A_f and I_f being the same quantities used in Eq. (4.2).

4.5 GENERATION OF STRUCTURE STIFFNESS MATRIX

The equilibrium conditions set up at the joints 'ij' (Fig. 4.4) of the frame yield

$$\left\{f_{ij}\right\}_{bi,j} + \left\{f_{ij}\right\}_{bi,j+1} + \left\{f_{ij}\right\}_{ci,j} + \left\{f_{ij}\right\}_{ci+1,j} = \left\{R_{ij}\right\} \qquad (4.5)$$

where $\left\{R_{ij}\right\}$ is the vector of external loads acting at the joint 'ij', $\left\{f_{ij}\right\}_{bij}$ and $\left\{f_{ij}\right\}_{bi,j+1}$ represent the forces at the end ij of the beams bij and bi+1,j respectively while $\left\{f_{ij}\right\}_{ci,j}$ and $\left\{f_{ij}\right\}_{ci+1,j}$ refer to the forces at the end ij of the columns $C_{i,j}$ and $C_{i+1,j}$ respectively. The member end forces of the beams and columns can be expressed in terms of the displacements at the ends of these members, using the respective member stiffness matrices (Eqs.4.3 and 4.4). Making use of Eqs. (4.3) and (4.4), Eq. (4.5) can be written as,

$$[\,G1\,]_j \left\{d_{i-1,j}\right\} + [\,G2\,]_j \left\{d_{i,j-1}\right\} + [\,G3\,] \left\{d_{ij}\right\}$$

$$+ [\,G4\,]_j \left\{d_{i,j+1}\right\} + [\,G5\,]_j \left\{d_{i+1,j}\right\} = \left\{R_{ij}\right\} \tag{4.6}$$

where G1 is obtained from the member stiffness matrix of the column c_{ij}, G5 from that of the column $c_{i+1,j}$, G2 from the member stiffness matrix of the beam b_{ij}, G4 from that of the beam $b_{i,j+1}$ while G3 is obtained from the stiffness matrices of the beams $b_{i,j}$ and $b_{i+1,j}$ and the columns c_{ij} and $c_{i+1,j}$ meeting at the joint 'ij'. The vectors $\left\{d_{i-1,j}\right\}$, $\left\{d_{i,j-1}\right\}$, $\left\{d_{ij}\right\}$, $\left\{d_{i+1,j}\right\}$ and $\left\{d_{i,j+1}\right\}$ represent the displacements θ, u and v of the joints (i-1,j), (i,j-1), (i,j), (i+1,j) and (i,j+1) respectively. G1 to G5 are square matrices of order 3 and the displacement vectors $\left\{d_{ij}\right\}$ etc. are of order (3 x 1). It is to be noted that the external forces $\left\{R_{ij}\right\}$ should also be expressed in nondimensionalised form as discussed in section 4.3. Equilibrium conditions generated for all the joints at the floor level 'i' yield,

$$[\,W\,]_i \left\{d_h\right\}_{i-1} + [\,X\,]_i \left\{d_h\right\}_i + [\,Y\,]_i \left\{d_h\right\}_{i+1} = \left\{R_h\right\}_i \tag{4.7}$$

$$W_i = \begin{bmatrix} [G1]_1 & & & & & & \\ & [G1]_2 & & & & & \\ & & - & & & & \\ & & & - & & & \\ & & & & - & & \\ & & & & & [G1]_{n-1} & \\ & & & & & & [G1]_n \end{bmatrix}_i$$

$$X_i = \begin{bmatrix} [G3]_1 & [G4]_1 & & & & \\ [G2]_2 & [G3]_2 & [G4]_2 & & & \\ & - & - & - & & \\ & & - & - & - & \\ & & & [G2]_{n-1} & [G3]_{n-1} & [G4]_{n-1} \\ & & & & [G2]_n & [G3]_n \end{bmatrix}_i$$

$$Y_i = \begin{bmatrix} [G5]_1 & & & & & & \\ & [G5]_2 & & & & & \\ & & - & & & & \\ & & & - & & & \\ & & & & - & & \\ & & & & & [G5]_{n-1} & \\ & & & & & & [G5]_n \end{bmatrix}_i$$

$$\{d_h\}_i = \begin{Bmatrix} d_{i,1} \\ d_{i,2} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ d_{i,n} \end{Bmatrix}$$

and

$$\{R_h\}_i = \begin{Bmatrix} R_{i,1} \\ R_{i,2} \\ \vdots \\ \vdots \\ R_{i,(n-1)} \\ R_{i,n} \end{Bmatrix}$$

In the development of the above equations, it could be observed that

$$[Y]_i = [W]^T_{i+j} \qquad (4.8)$$

where the superscript 'T' refers to the transpose of the matrix. The matrices W, X and Y are of order (3n x 3n) and vectors $\{d_h\}$ and $\{R_h\}$ are of order (3n x 1). Similar to Eq. (4.7) equations of equilibrium can be written up for all the joints in each of the floor levels 1 to 'm' as

$$[K_k] \{d_k\} = \{R_k\} \qquad (4.9)$$

where the structure stiffness matrix

$$K_k = \begin{bmatrix} X_1 & Y_1 & & & & & \\ W_2 & X_2 & Y_2 & & & & \\ & W_3 & X_3 & Y_3 & & & \\ & & - & - & - & & \\ & & & - & - & - & \\ & & & & W_{m-1} & X_{m-1} & Y_{m-1} \\ & & & & & W_m & X_m \end{bmatrix}$$

$$d_k = \begin{Bmatrix} d_{h1} \\ d_{h2} \\ \vdots \\ d_{hm} \end{Bmatrix}$$

$$R_k = \begin{Bmatrix} R_{h1} \\ R_{h2} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ R_{hm} \end{Bmatrix}$$

The boundary conditions corresponding to the fixed base of the frame require that the displacements of the joints at the first floor (i.e.) ground level (Fig.4.1) $\{d_{h1}\} = 0$. Incorporating these boundary conditions in Eq. (4.9), the final set of equilibrium equations for the frame boils down to,

$$\begin{bmatrix} X_2 & Y_2 & & & & & & \\ W_3 & X_3 & Y_3 & & & & & \\ & - & - & - & & & & \\ & & - & - & - & & & \\ & & & & W_{m-1} & X_{m-1} & Y_{m-1} \\ & & & & & W_m & X_m \end{bmatrix} \begin{Bmatrix} d_{h2} \\ d_{h3} \\ \vdots \\ \vdots \\ d_{h(m-1)} \\ d_{hm} \end{Bmatrix} = \begin{Bmatrix} R_{h2} \\ R_{h3} \\ \vdots \\ \vdots \\ R_{h(m-1)} \\ R_{hm} \end{Bmatrix} \tag{4.10}$$

4.6 EVALUATION OF INTERACTION FORCES

It can be observed that upto the development of Eq. (4.10), the formulation is quite identical to that in the finite element method discussed in Chapter 3. For the interaction analysis of frame and shear wall system using the iterative scheme (discussed in Chapter 2), the interaction forces on the frame at the points of connection with the shear wall are to be computed, when the frame is subjected to a known set of displacements (supplied by the shear wall analysis) at these connection points. The interaction forces are obtained by suitably modifying Eq. (4.10) as explained below. It is assumed that the shear wall is situated on the right hand side of the frame (along the column line 'n' in Fig. 4.1). Hence the displacements of the joints on the column line 'n' would be known and the interaction forces at these joints of the frame corresponding to the above displacements are to be evaluated.

W, X, Y, d_h and R_h matrices in Eq. (4.10) are partitioned and expressed in the following manner.

$$W_i = \begin{bmatrix} W1 & W2 \\ W3 & W4 \end{bmatrix}_i \qquad (4.11)$$

$$X_i = \begin{bmatrix} X1 & X2 \\ X3 & X4 \end{bmatrix}_i \tag{4.12}$$

$$Y_i = \begin{bmatrix} Y1 & Y2 \\ Y3 & Y4 \end{bmatrix}_i \tag{4.13}$$

$$\left\{d_h\right\}_i = \left\{\begin{array}{c} d_{i,1} \\ d_{i,2} \\ \vdots \\ d_{i,n-1} \\ \hline d_{i,n} \end{array}\right\} = \left\{\begin{array}{c} d_{pi} \\ d_{in} \end{array}\right\} \tag{4.14}$$

and

$$\left\{R_h\right\}_i = \left\{\begin{array}{c} R_{i,1} \\ R_{i,2} \\ \vdots \\ R_{i,n-1} \\ \hline R_{i,n} \end{array}\right\} = \left\{\begin{array}{c} R_{pi} \\ R_{i,n} \end{array}\right\} \qquad (4.15)$$

In Eqs. (4.11) to (4.13), W1, X1 and Y1 represent partitioned matrices of order (3n-3) x (3n-3), W2, X2 and Y2 represent rectangular matrices of order (3n-3) x 3, W3, X3 and Y3 are matrices of order 3 x (3n-3) while W4, X4 and Y4 are square matrices of order 3. Incidentally, it can be noticed that W2, W3, Y2 and Y3 are null matrices. In Eqs. (4.14) and (4.15), d_{pi} and R_{pi} are partitioned matrices of order (3n-3) x 1 and represent the displacements and applied external loads respectively at the joints 1 to (n-1) on the i-th floor.

Substituting Eqs. (4.11) to (4.15) in Eq.(4.7), the equations of equilibrium for the joints on the i-th

But $[W2]$ and $[Y2]$ are null matrices. So,

$$F_{p,i} = R_{p,i} - [X2]_i \{d_{i,n}\} \tag{4.18}$$

Eq. (4.17) represents the modified set of equations of equilibrium for the joints on the i-th floor, involving only the known external forces and unknown joint displacements on this floor. Similar expressions can be developed from Eq. (4.10) for the joints in the floor levels 2 to m and written as

$$\begin{bmatrix} |X1|_2 & |Y1|_2 & & & & \\ |W1|_3 & |X1|_3 & & & & \\ - & - & - & & & \\ & - & - & - & & \\ & & |W1|_i & |X1|_i & |Y1|_i & \\ & & & - & - & - \\ & & & & |W1|_m & |X1|_m \end{bmatrix} \begin{Bmatrix} d_{p2} \\ d_{p3} \\ , \\ , \\ d_{pi} \\ , \\ d_{pm} \end{Bmatrix} = \begin{Bmatrix} F_{p2} \\ F_{p3} \\ , \\ , \\ F_{pi} \\ , \\ F'_{pm} \end{Bmatrix} \tag{4.19}$$

Using the scheme of forward elimination and backward substitution (58), Eq. (4.19) is solved for the displacement vectors d_{p2} to d_{pm}.

Again from Eq. (4.16) representing the equilibrium of the joints of the frame on the 'i'-th floor, the interaction force,

$$\begin{aligned} R_{i,n} = & [W3] \{d_{p,i-1}\} + [W4] \{d_{i-1,n}\} \\ & + [X3] \{d_{p,i}\} + [X4] \{d_{i,n}\} \\ & + [Y3] \{d_{p,i+1}\} + [Y4] \{d_{i+1,n}\} \end{aligned} \tag{4.20}$$

But W3 and Y3 are null matrices.

So,

$$\begin{aligned} \{R_{i,n}\} = & [W4] \{d_{i-1,n}\} + [X3] \{d_{p,i}\} \\ & + [X4] \{d_{i,n}\} + [Y4] \{d_{i+1,n}\} \end{aligned} \tag{4.21}$$

In Eq. (4.21), the displacement vectors $\{d_{i,n}\}$, $\{d_{i-1,n}\}$ and $\{d_{i+1,n}\}$ are known from the shear wall analysis and d_{pi} is obtained as the solution of Eq. (4.19) and hence the interaction force vector $R_{i,n}$ is computed. In the same way, the interaction forces on the frame at all floor levels are evaluated. When the frame is connected

to the shear wall through beams at floor levels and no columns exist at the junction of frame and wall, the analysis for the interaction forces on the frame can be carried out as discussed above by assigning zero values for the area of cross-section and second moment of area of the missing columns.

4.7 TESTING COMPUTER PROGRAMS

Two computer programs were developed using the method of analysis of frames discussed in the previous section. The first program is used to analyse a rectangular frame for joint displacements and member end forces, when the loads applied at the joints are known. The second program is capable of computing the interaction forces on the frame in a shear wall-frame system, when the displacements of the frame joints connected to the shear wall are known. The first program has been developed, mainly to help check the correctness of the latter. Both the above programs require memory locations that are functions of the number of bays in the frame rather than the number of storeys. Very tall building frames could be analysed without requiring large core memory space in the computer, provided auxiliary storage units

like tapes or discs are available. The first program was checked by comparing the member end moments for a two bay two storey frame, obtained by the above program with those obtained by using Kani's method (60) of rotation contributions. The member end moments tallied with a maximum difference of about 0.5 percent. The member end forces obtained by using the above program satisfied the joint equilibrium conditions perfectly well. Thus the reliability of the first program was ascertained.

A two bay 10 storeyed frame (representing a typical frame interconnected to a shear wall) was subjected to external loads as shown in Fig. 4.5. The cross-sectional properties of all columns were equal and so was the case for the beams in the frame. The properties of beam and column sections are listed in Table 4.1. The displacements viz. rotations, lateral and vertical translations of the frame joints wherein the external loads act were computed using the first program. These displacements were fed as data for the second program and the interactions forces required to subject the frame (4.5) to this set of displacements were evaluated. The loads used in the first program and the interaction forces computed by the use of the second

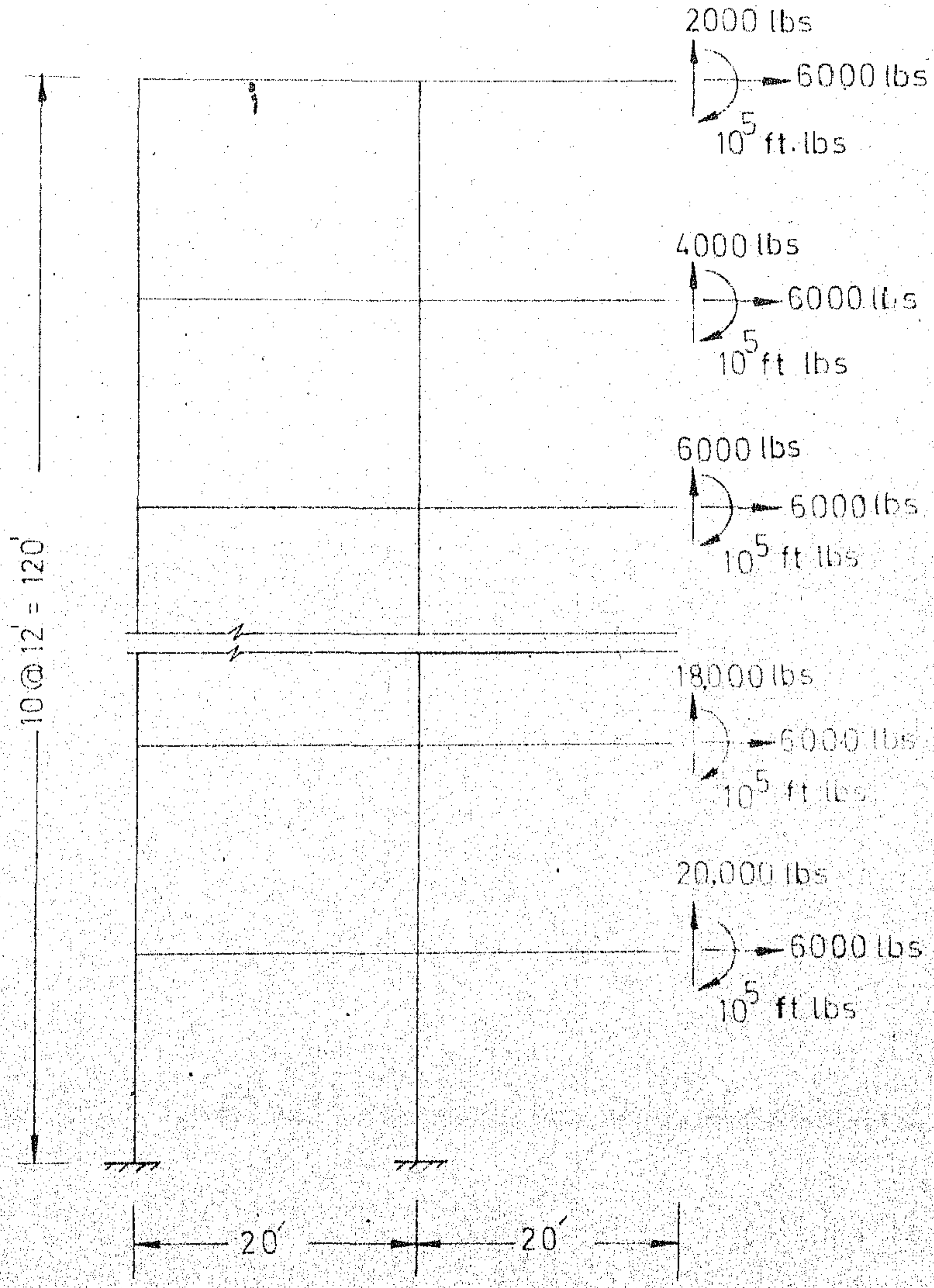


FIG.4·5 TEN STOREY FRAME

TABLE 4.1

CROSS - SECTIONAL PROPERTIES

OF MEMBERS OF FRAME

Member	Span	Area of X-ion	Moment of inertia
Beam	20'-0"	200 sq.in.	6670 $in.^4$
Column	12'-0"	180 sq.in.	4860 $in.^4$

TABLE 4.2

COMPARISON OF FORCES APPLIED IN

PROGRAM 1 AND COMPUTED FROM PROGRAM 2

Floor level	Moments (ft.lbs) Data in 1st prog.	Result in 2nd prog.	Lateral force (lbs) Data in 1st prog.	Result in 2nd prog.	Vertical force (lbs) Data in 1st prog.	Result in 2nd prog.
2	100000	99999.86	6000	6000.07	20000	19999.99
3	100000	100000.08	6000	6000.40	18000	18000.01
4	100000	99999.97	6000	6000.46	16000	16000.00
5	100000	99999.92	6000	6000.76	14000	13999.99
6	100000	100000.05	6000	6001.00	12000	12000.00
7	100000	99999.84	6000	6000.16	10000	10000.00
8	100000	100000.17	6000	6001.23	8000	8000.01
9	100000	99999.89	6000	6001.23	6000	5999.99
10	100000	100000.10	6000	6001.23	4000	4000.01
11	100000	99999.91	6000	6000.52	2000	2000.00

program are presented in Table 4.2. The agreement between the quantities listed in the Table 4.2 ensures the correctness of the program developed for the evaluation of the interaction forces on the frame.

CHAPTER 5

INTERACTION BEHAVIOUR AND DESIGN CURVES

5.1 GENERAL

The structural system consisting of frame and the interconnected shear wall is analysed using the iterative scheme discussed in Chapter 2. A detailed study of the interaction behaviour and influences of various parameters on the interaction forces between the wall and the frame, carried out using the iterative method is discussed in this chapter. As the shear wall and frame are treated as separate entities, the sizes of matrices dealt with are relatively small and consequently the arithmetic operations involve less error.

Study of the individual behaviours of the frame and wall in the shear wall-frame system under the action of total applied loads, indicates the effectiveness of the interconnected system in restricting the lateral deflections. In the 10 storey shear wall-frame structure in Fig. 5.1, the lateral loads are applied on the frame alone and the lateral displacements of the frame at the floor levels are computed. Next, all the lateral loads are applied on the shear wall only and the corresponding

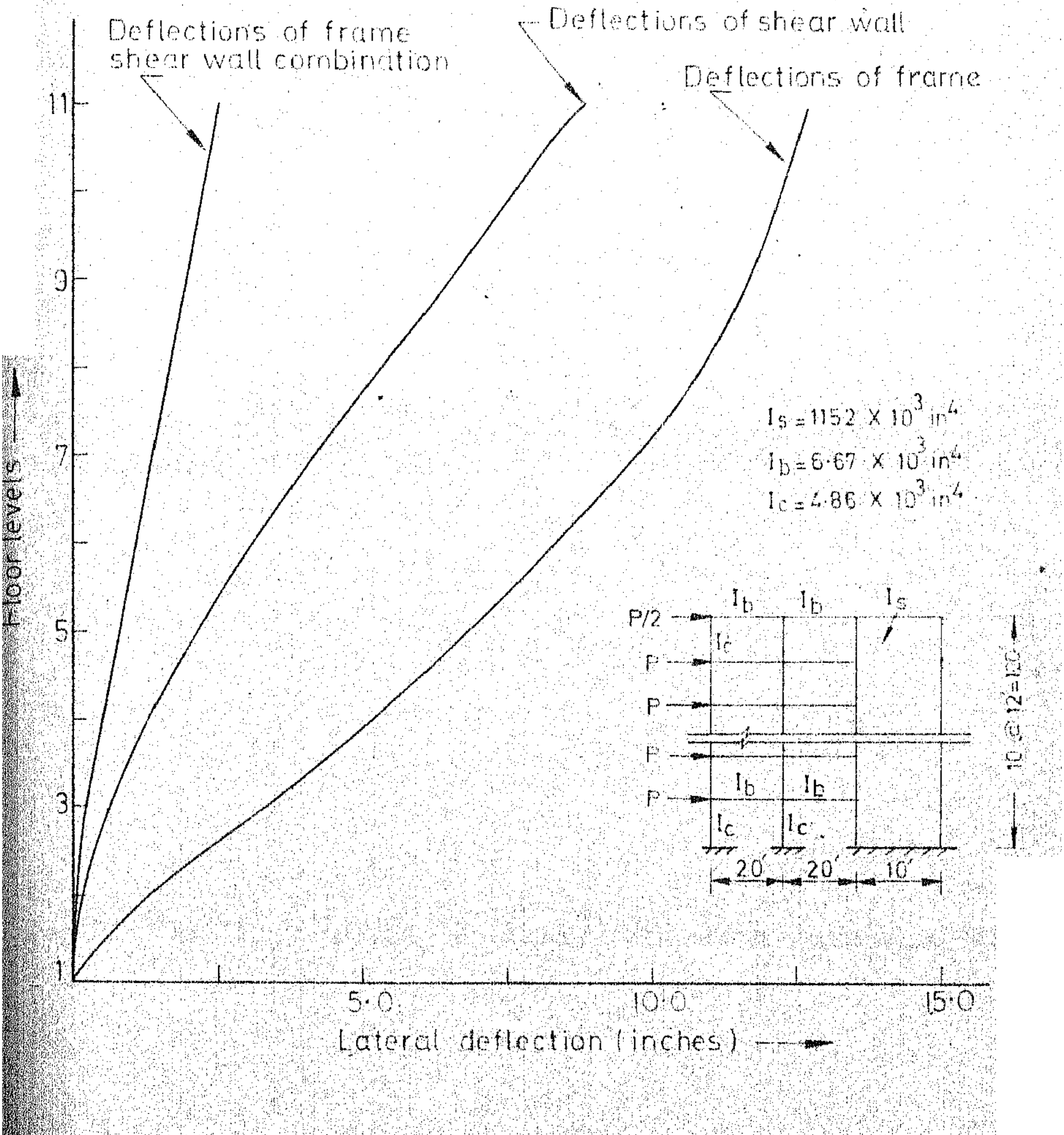


FIG. 51 10 STOREY FRAME SHEAR WALL DEFLECTIONS

lateral displacements at the floor levels are evaluated by the conjugate beam method. Finally the lateral loads are applied on the interconnected frame wall system and the lateral displacements of the points of connection between the wall and frame are calculated by the iterative analysis. The above computed lateral displacements of the frame, wall and the interconnected frame and wall are plotted in Fig. 5.1. It can be observed that the combination of shear wall and frame restricts the lateral displacements at different floor levels quite effectively.

The portion of the applied lateral loads carried by the frame in the shear wall-frame system is mainly a function of the relative stiffness of wall to that of the frame. When the shear wall is relatively slender, the frame also carries a considerable part of the total applied lateral loads. Fig. 5.3 indicates the share of the applied lateral loads, carried by a 20 storey two-bay frame (Fig. 5.2) for two different stiffnesses of the interconnected shear wall. In both cases, the shear wall has constant cross-section along the height. The moments of inertia of the columns and beams decrease uniformly in each of the storeys, with their top storey values being one tenth and one half of their first storey values

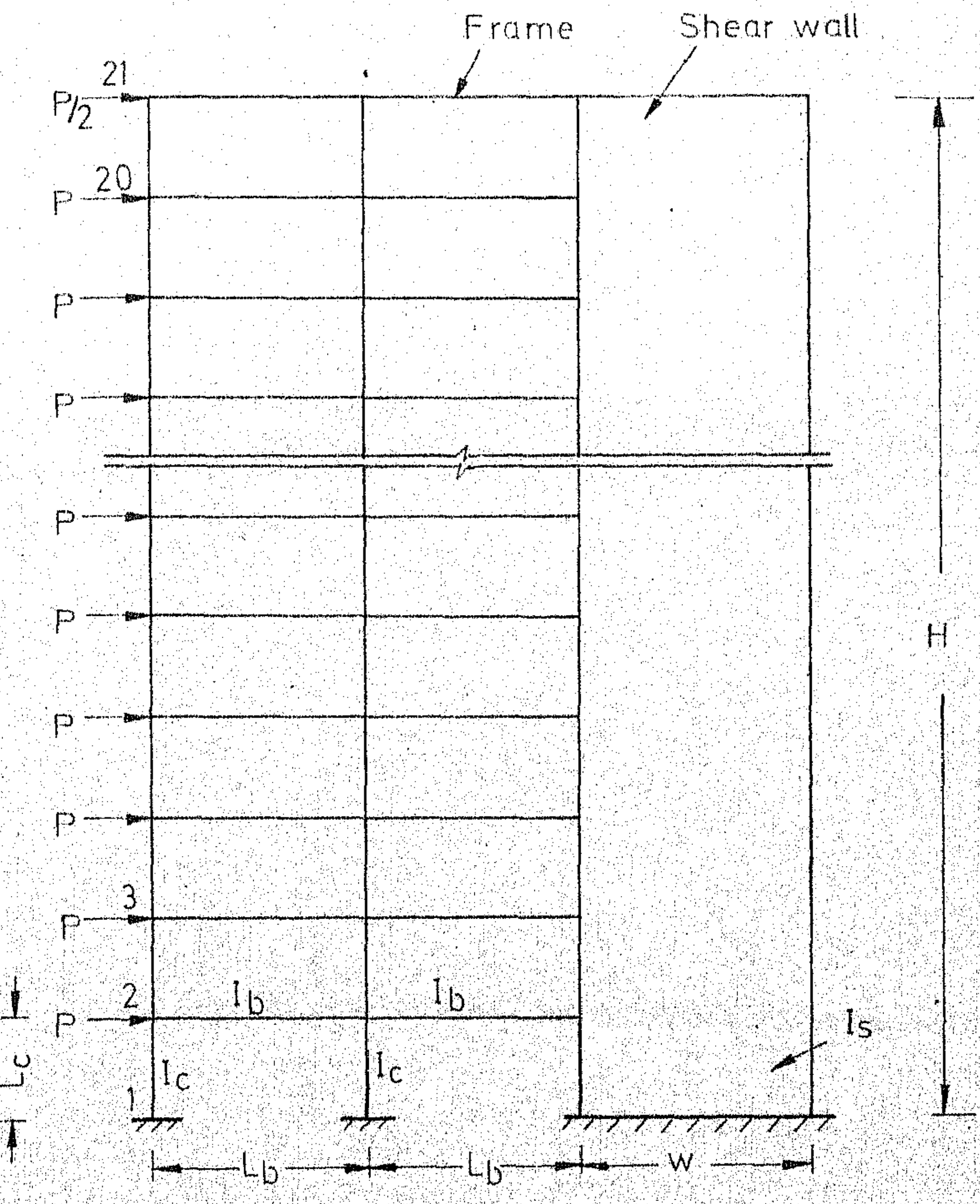


FIG. 5·2 20 STOREY SHEAR WALL FRAME STRUCTURE

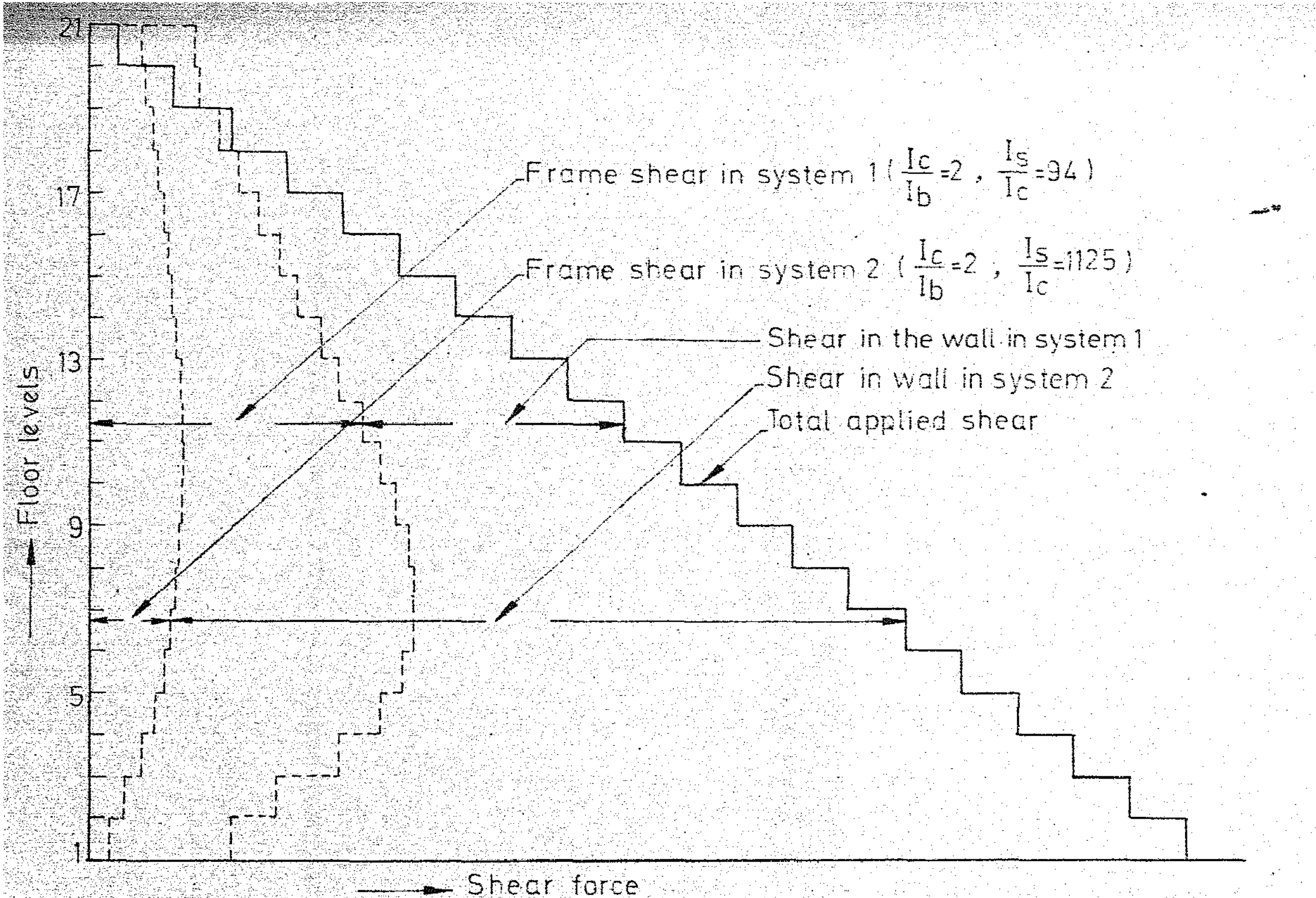


FIG. 5.3 SHEAR FORCE DISTRIBUTION IN TWO 20 STOREY FRAME WALL-SYSTEMS

respectively. In the wall-frame system 1, the shear wall is a slender one with its moment of inertia about 94 times greater than that of the column in the first storey. In system 2, the frame is the same as in system 1 but the wall is much stiffer with its moment of inertia about 1100 times greater than that of the column in the first storey. The plot of the storey shears in the frames in system 1 and system 2 indicates that the frames, carry a substantial portion of the lateral loads applied on the frame and wall system and this share increases with the increase in the relative stiffness of the frame.

5.2 COMPUTER PROGRAM FOR INTERACTION ANALYSIS

The computer program for the interaction analysis of frame and shear wall structure has been prepared by combining the programs developed for the displacement analysis of shear wall using the conjugate beam method and that for the frame employing the stiffness method, discussed in Chapters 3 and 4 respectively. This program is used to analyse a 20 storey 3 bay frame and an interconnected shear wall (Fig. 5.4). The wall has constant moment of inertia along its height. The moments of inertia of beams and columns vary for every two storeys with their

$$\frac{L_b}{L_c} = 2\cdot5 \;; \quad \frac{I_c}{I_b} = 2 \;; \quad \frac{I_s}{I_c} = 1500$$

$$\left(\frac{ns}{20}\right)^2 \quad \frac{\Sigma I_c}{\Sigma I_s} = 0\cdot004 \;; \quad \frac{I_c}{I_b} \times \frac{L_b}{L_c} = 5$$

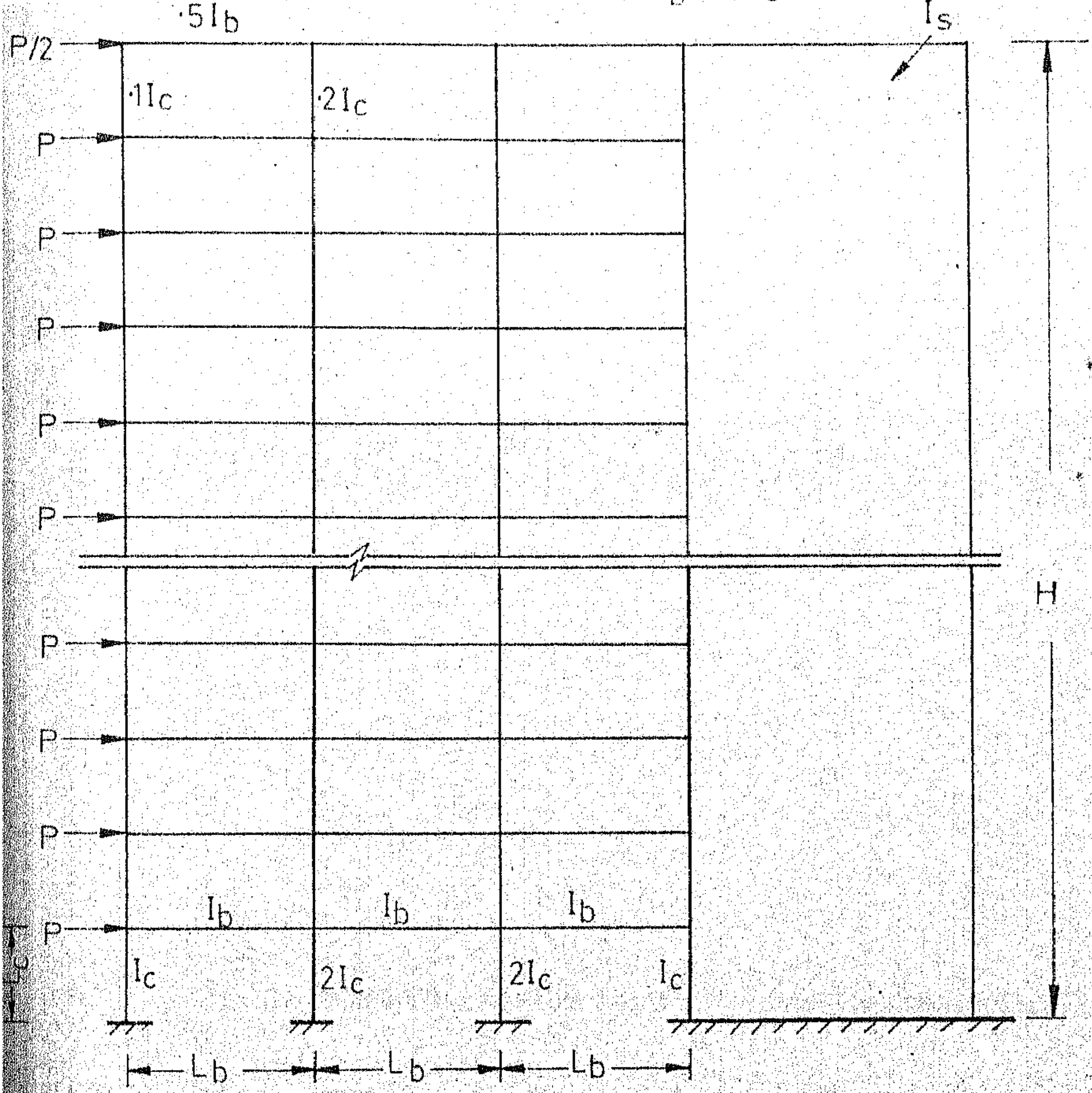


FIG. 5.4 20 STOREY SHEAR WALL-FRAME STRUCTURE (FOR COMPARISON WITH PARME'S ANALYSIS)

first storey and top storey values as indicated in Fig. 5.4. The interaction forces between the wall and the frame for such frame-wall systems have been presented in graphical form by Parme (32). The graphs give the ratios of storey shears in the frame to the corresponding total applied shear in that storey and the bending moments in the wall at different levels as a fraction of the moment at the base of the wall caused by the applied loads. However, the method of analysis employed by Parme involved certain approximations and hence the results of the analysis of the frame-wall system (Fig. 5.4) by the present iterative scheme can be expected to compare only with some tolerance with the graphical values presented by Parme. The data pertaining to the shear wall-frame system analysed are indicated in Fig. 5.4. Parme has not considered the effect of elongation or contraction of the outer fibres of the wall on the girder end moments. Hence suitable modifications have been introduced in the computer program to discard this effect. The ratios of storey shears in the frame and the moments on the wall calculated using the computer program and obtained from the graphs presented by Parme (32), are listed in Table 5.1. It is seen that there is good agreement between the computed and graphical values in the case of moments on

TABLE 5.1

COMPARISON OF FRAME SHEAR FORCES AND

WALL MOMENTS IN 20 STOREY FRAME WALL SYSTEM

Position along height from base	Frame Shear Forces (V_{fx}/V_{tx})		Wall Moments (M_{wi}/M_b)	
	Graph	Computed	Graph	Computed
0	0.050	0.032	0.800	0.790
.1 H	0.087	0.073	0.625	0.610
.2 H	0.150	0.117	0.470	0.460
.3 H	0.200	0.155	0.340	0.330
.4 H	0.250	0.190	0.230	0.225
.5 H	0.300	0.226	0.140	0.142
.6 H	0.350	0.268	0.075	0.079
.7 H	0.425	0.324	0.030	0.034
.8 H	0.580	0.414	Not	0.006
.9 H	1.020	0.550	Available	-0.004
.95 H	1.950	2.120	-	-0.005

the wall at different levels. The values of the storey shears match only with a wide tolerance.

5.3 CONVERGENCE STUDY

The accuracy of the results of the interaction analysis of shear wall and frame system employing the iterative scheme can be improved upon by specifying strict convergence criteria. Specification of about 5 percent tolerance for the convergence of the initial and end displacements in the iteration cycle offers reliable results for all practical purposes. The rate of convergence of the iterative scheme to the correct solution is quite fast for stiffer walls, when the displacements corresponding to the free deflected shape of the wall are used as the initial values. The normal procedure of using the end values of the previous iteration cycle for the initial values of the next cycle proved futile in many cases with divergent results, as a proper initial deflected shape could not be obtained by guess. Two extrapolation techniques have been tried to obtain the initial values for any iteration cycle based on the values of the previous iteration cycles, and hence improve the convergence rate.

'Forced convergence extrapolation' technique suggested by Khan and Sbarounis (29) consists of the evaluation of the initial values of lateral and rotational displacements for any cycle other than the first, based on the displacements of the previous cycle and the free displacements (d_f) of the shear wall. The extrapolation formula is expressed as

$$d_{i,(n+1)} = d_{i,n} + \frac{d_{e,n} - d_{i,n}}{1 + \dfrac{d_f - d_{e,n}}{d_{i,n}}} \qquad (5.1)$$

where 'd' refers to either lateral or rotational displacements at any particular floor level, the first subscripts 'i' and 'e' refer to the initial and end values respectively in the iteration cycle, and the number of the cycle is indicated by the second subscript. However, for the vertical displacements, the end values of the previous cycle are used as the initial values in the next cycle.

Aitken's δ^2 extrapolation technique (61) developed for use in the iterative methods of solving non-linear equations, requires the end values of the previous three iteration cycles for the estimation of the initial values of the next cycle. The mathematical expression for the above extrapolation technique is

$$d_{n+3} = d_{n+2} - \frac{\left[d_{n+2} - d_{n+1} \right]^2}{d_{n+2} - 2d_{n+1} + d_n} \qquad (5.2)$$

This extrapolation technique can be used only after the first three iteration cycles have been carried out.

To evaluate the relative efficiencies of the above two extrapolation methods, both were used in the iterative scheme for the interaction analysis of 10 storey and 20 storey shear wall-frame structures. A tolerance limit of 5 percent was employed as the convergence criterion. In the case of 10 storey two bay frame with an interconnected shear wall, the ratio of the second moment of area of the shear wall to that of the column was about 240. The forced convergence technique required 10 iterations and the Aitken's δ^2 extrapolation required 4 iterations for convergence. In one case of a 20 storey two bay frame with an interconnected shear wall, the ratio of the second moment of area of the wall to that of the column in the first storey was 1120. For this structure, the forced convergence technique as well as the Aitken's δ^2 extrapolation required 4 iterations for convergence. For another case of 20 storey shear wall and frame structure in which the shear wall was relatively slender

with its second moment of area only 95 times greater than that of the column, the forced convergence technique required 12 iterations. But the results of the iterative scheme using Aitken's δ^2 extrapolation technique continued to diverge even for the tenth iteration with no trend of convergence in any further cycles. Similar divergent behaviour of the results was met with in another case of slender shear wall interconnected to a frame when Aitken's δ^2 extrapolation was used. But the forced convergence technique offered converged results for the above structure. Based on this, it has been decided to use the forced convergence technique for the interaction analysis of shear wall-frame structures considered in this work.

5.4 EFFECT OF VARIATION OF STIFFNESS OF THE MEMBERS OF THE FRAME ALONG THE HEIGHT

Three different systems of 20 storey frame with an interconnected shear wall (Fig. 5.2) were analysed to investigate the effect of variation of the flexural stiffnesses ($I/_L$) of the members of the frame along height. In all the three cases, the shear wall had the same moments of inertia and the cross-sections of the walls were constant along the height. In system 1, the moments of

inertia of the columns and beams decreased uniformly in every storey, starting from bottom storey as shown in Figs. 5.5a and 5.6a. In system 2, the decrease in the moments of inertia of the columns and beams occurred for every two storeys (Figs. 5.5b and 5.6b). In system 3, there was no variation in the values of the moments of inertia of the columns and beams and they were equal in all the storeys as in the first storey.

Figs. 5.7 and 5.8 give the plot of the lateral interaction forces (F_f), (discrete forces at the floor levels where the frame is connected to the shear wall) as a fraction of the total applied loads (ΣP) on the structure. The lateral interaction forces shoot upto very high values at the top floor levels and are not indicated in the plot except by their numerical values. It can be noticed that in the cases of uniform and no variation of the moments of inertia of the members of the frame, the wall is pulled towards the frame in the upper storeys and the reverse takes place in the lower storeys (Fig.5.7). But in the case of the variation of moments of inertia of the beam and column members occurring in alternate storeys, the lateral interaction forces are distributed in a non-uniform pattern, especially in the upper storeys

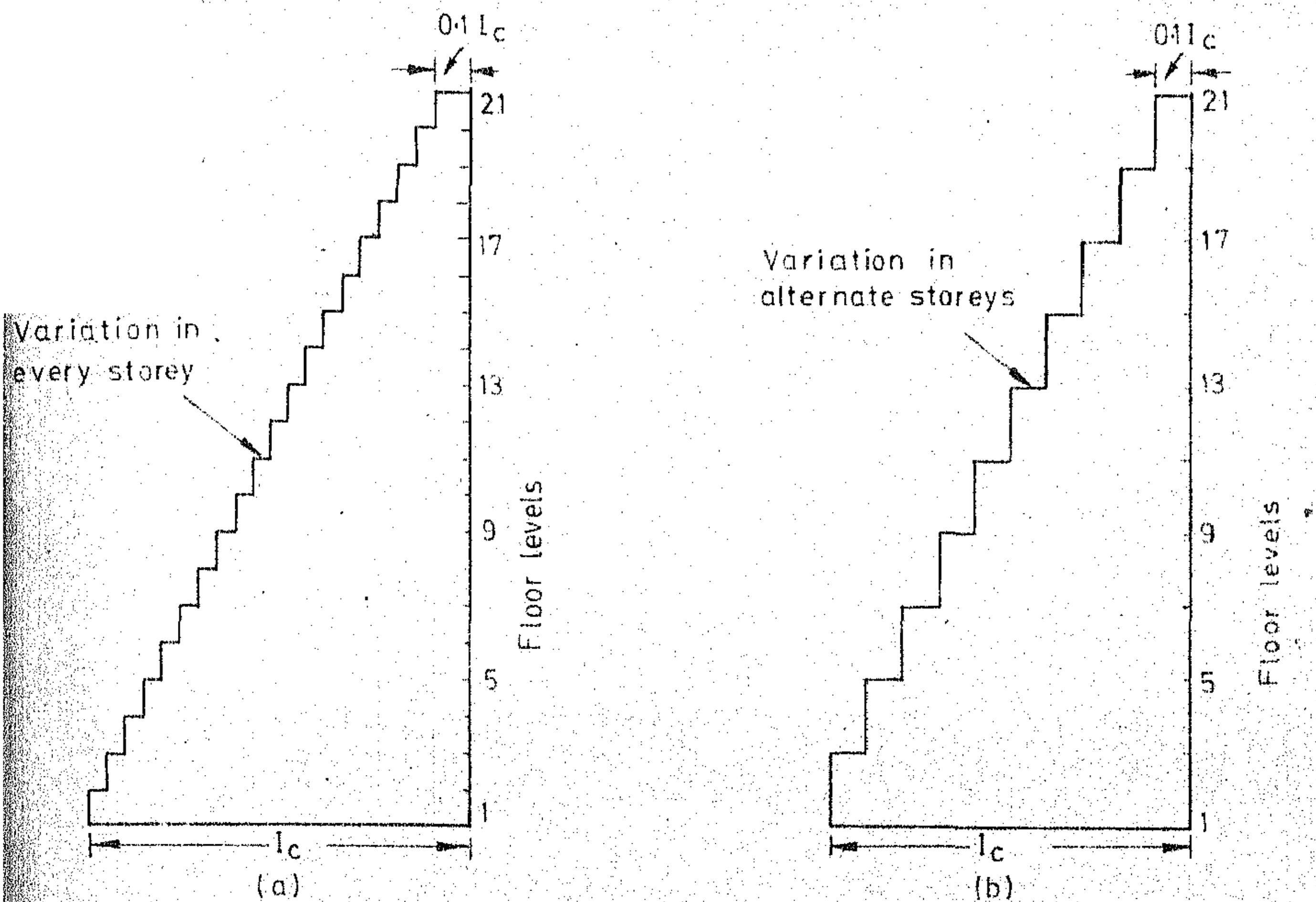


FIG. 5·5 VARIATION OF M·I OF COLUMN ALONG HEIGHT

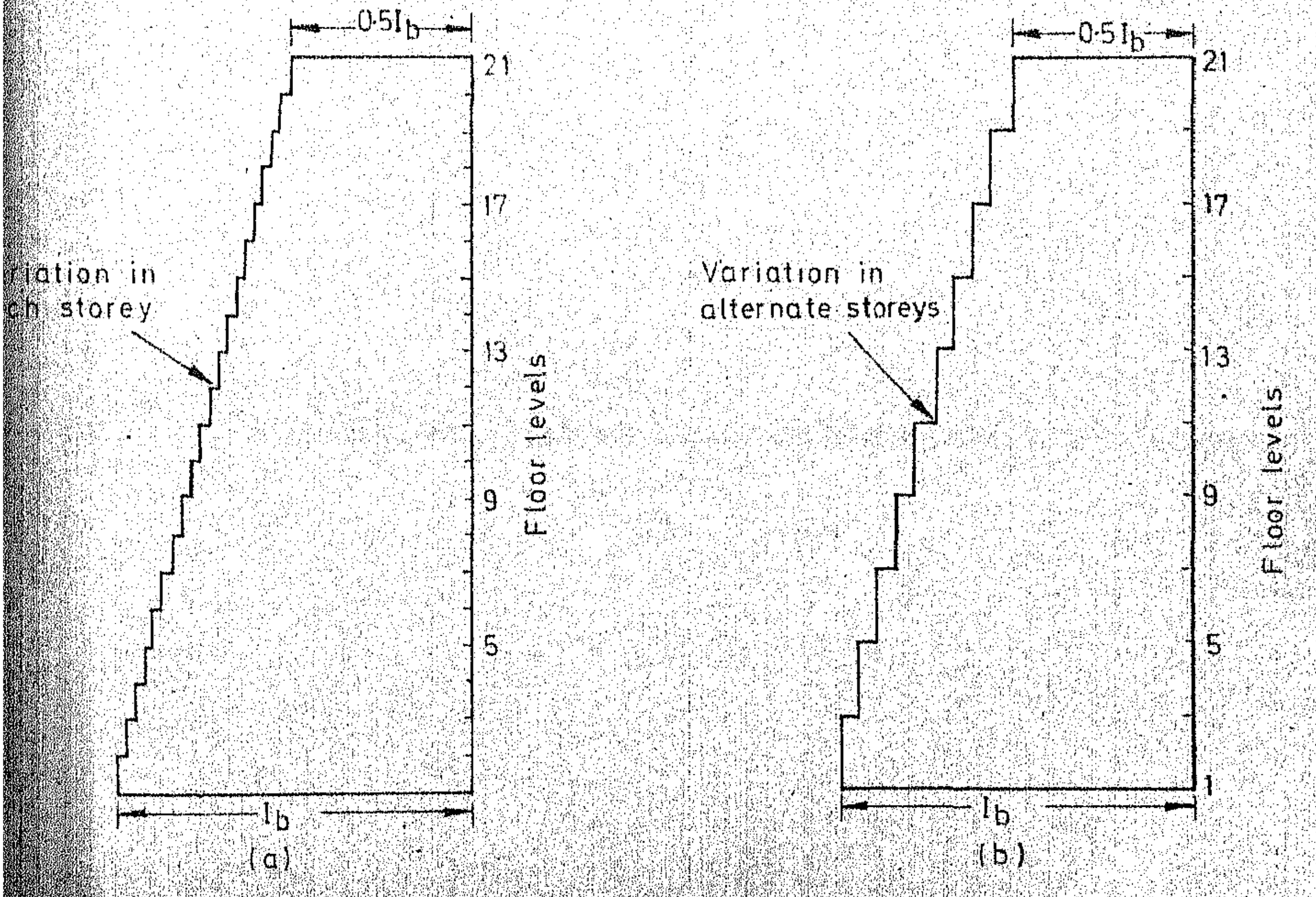


FIG. 5·6 VARIATION OF M·I OF BEAM ALONG HEIGHT

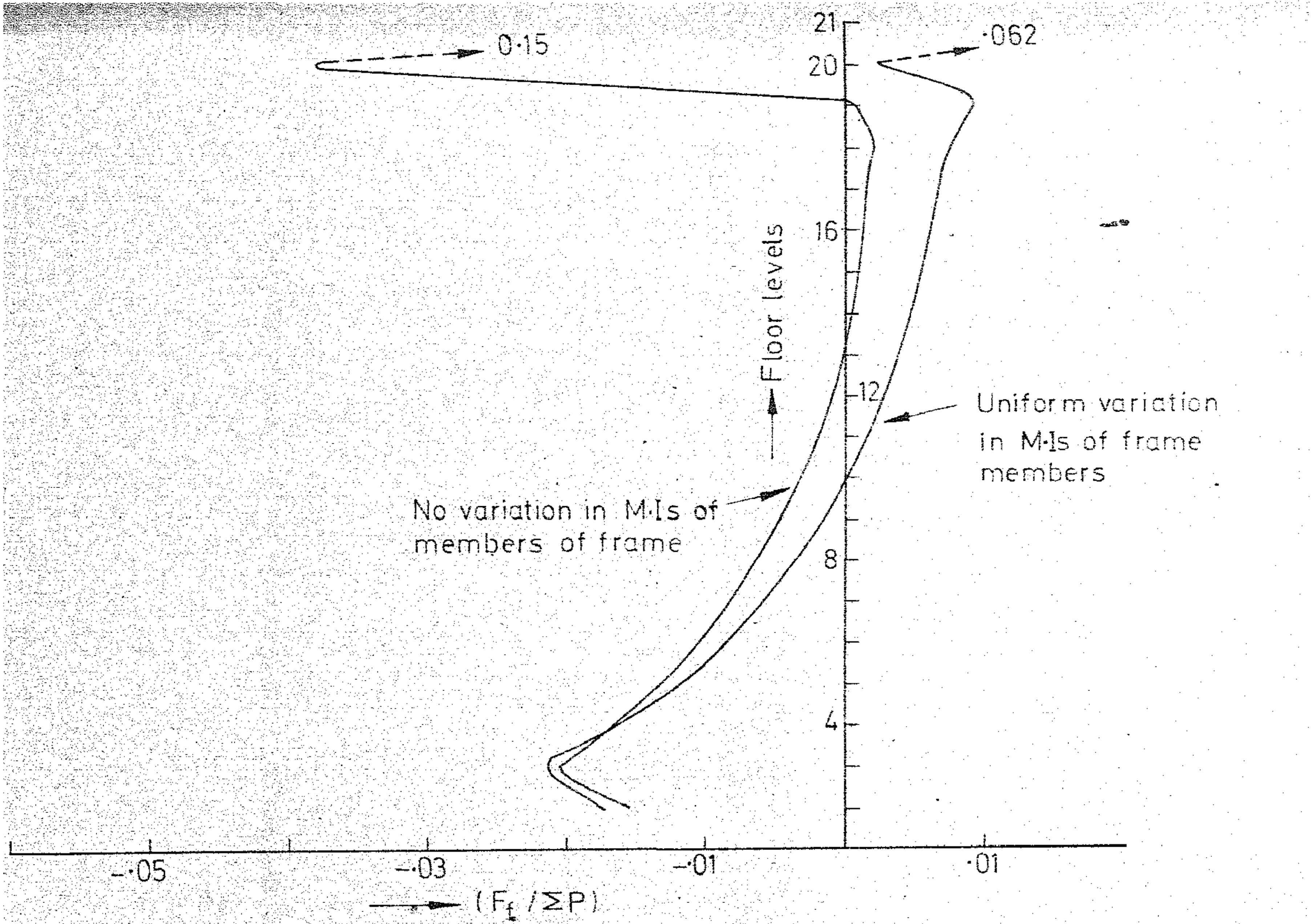


FIG.5·7 LATERAL INTERACTION FORCES IN 20 STOREY FRAME-WALL SYSTEM ON FRAME

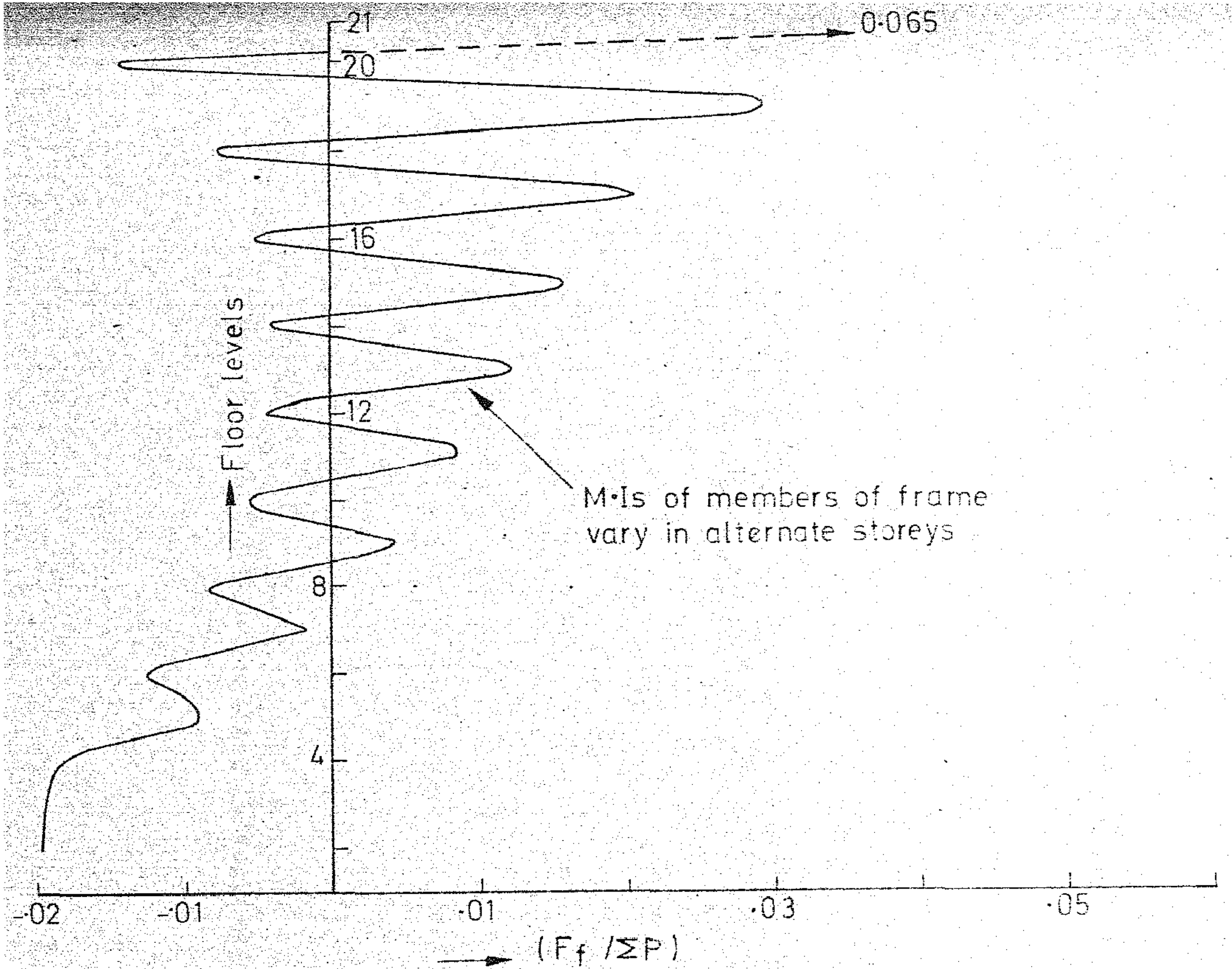


FIG.5·8 LATERAL INTERACTION FORCES IN 20 STOREY FRAME-WALL SYSTEM ON FRAME

where their directions change in alternate floor levels. This indicates that the lateral interaction forces are quite sensitive to the variations in the stiffnesses of the members of the frame along the height.

Further, the shooting up of the interaction forces in the top storey has been reported by some investigators (29, 32). The increase in the interaction force values at the top floor is much higher for a frame with no variation in the moments of inertia of the members of the frame when compared to the frame with variations along the height. However, the interaction moments on the frame do not differ much for systems 1, 2 and 3.

5.5 EFFECT OF VARIATION OF THE MOMENT OF INERTIA OF THE SHEAR WALL ALONG THE HEIGHT

To find the influence of the variation of the moments of inertia of the wall in the shear wall-frame system on the interaction behaviour, four 20 storey shear wall-frame structures (Fig. 5.2) were analysed. In each of the four systems, the frames were identical to each other and the moments of inertia of the walls in the first storey were equal. In system 1, the wall had constant cross-section all along the height. In

system 2, the moment of inertia of the wall in the top storey was 4/10 times that in the first storey and the reductions in the moment of inertia took place in each of the storeys (Fig. 5.9a). In system 3, the moments of inertia of the wall in the top storey was the same as in system 2 but the decrease in the moments of inertia occurred for every two storeys (Fig. 5.9b). In system 4, the decrease in the moments of inertia of the wall occurred for every four storeys (Fig. 5.9c). The results of the interaction analysis for the above shear wall-frame systems indicate that moderate reductions along the height in the moments of inertia of the shear wall do not affect the interaction behaviour in any appreciable manner. It is uneconomical to provide shear walls with constant cross-sections without any reductions along the height, except for specific reasons.

5.6 INFLUENCE OF THE AXIAL DEFORMATIONS OF THE WALL

With the aim of simplification of the interaction analysis of shear wall-frame structures, some investigators (27, 32) have neglected the effect of axial deformations of the walls caused by the flexural rotations and

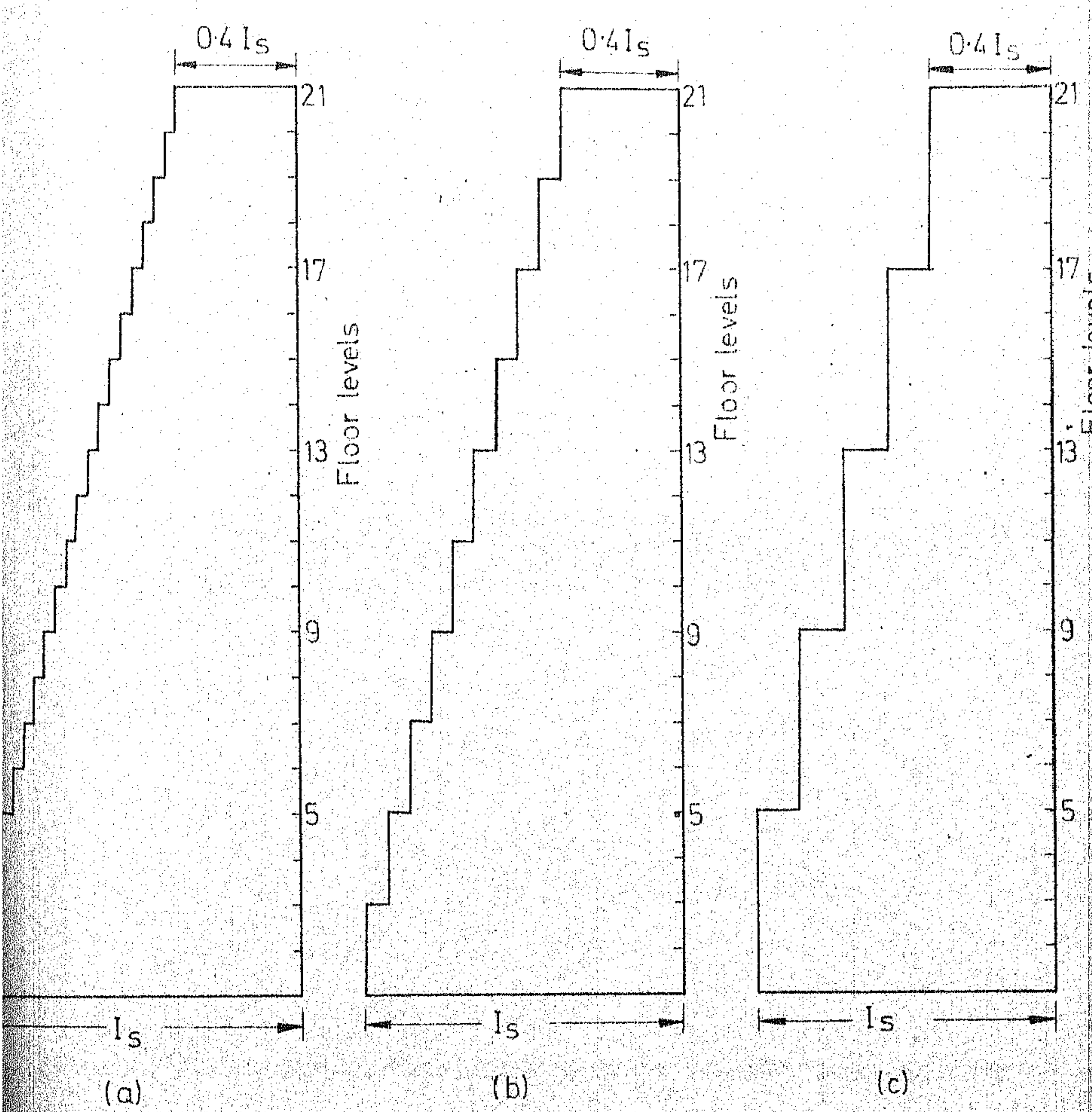


FIG. 5·9 VARIATION OF M·I OF SHEAR WALL ALONG HEIGHT

axial loads, on the girder end moments of the frame. To estimate the amount of approximations involved in neglecting the effect of axial deformations of the wall on the girder end moments and in disregarding the vertical interaction forces in the computation of moments at different levels of the wall, four cases of 20 storey frames with interconnected shear wall (Fig. 5.2) were considered. In all the four systems, the frames were identical but the shear walls had different stiffnesses. The ratios of the moments of inertia of the wall to that of the column in the first storey were 95 in system 1, 375 in system 2, 1125 in system 3 and 1880 in system 4. The shear walls had constant cross section along the height in each of the four systems. For all the four systems of shear wall-frame structure, the interaction analyses were carried out, first including the effects of axial deformations of the walls and next, by neglecting the effects of axial deformations. The results of the analyses for two typical cases are presented in Tables 5.2 and 5.3. The tables give the values of the interaction moments (M_f) on the frame and the moments on the wall (M_w) at different floor levels as a fraction of the moment at the base (M_b) of the structure due to the applied loads. In general, it is observed that neglecting axial deformations

TABLE 5.2

COMPARISON OF INTERACTION MOMENTS

ON FRAME AND MOMENTS ON WALL

$$\frac{I_c}{I_b} = 2; \ \frac{L_c}{L_b} = .376; \ \frac{I_s}{I_c} = 1125$$

Height from base	Interaction Moments On Frame (M_f /M_b)		Moments On Shear Wall (M_w /M_b)	
	With axial deflections of wall	Neglecting axial deflections of wall	With axial deflections of wall	Neglecting axial deflections of wall
.1 H	.00154	.00132	.602	.638
.2 H	.00256	.00220	.448	.481
.3 H	.00318	.00273	.320	.350
.4 H	.00348	.00300	.216	.241
.5 H	.00355	.00308	.133	.154
.6 H	.00346	.00300	.070	.086
.7 H	.00325	.00283	.026	.037
.8 H	.00296	.00258	-.0005	.0071
.9 H	.00261	.00228	-.0101	-.0059
1.0 H	.00202	.00174	-.0028	-.0017

TABLE 5.3

COMPARISON OF INTERACTION MOMENTS

ON FRAME AND MOMENTS ON WALL

$$\frac{I_c}{I_b} = 2; \quad \frac{L_c}{L_b} = .376; \quad \frac{I_s}{I_c} = 95$$

Height from base	Interaction Moments On Frame (M_f / M_b)		Moments On Shear Wall (M_w / M_b)	
	With axial deflections of wall	Neglecting axial deflections of wall	With axial deflections of wall	Neglecting axial deflections of wall
.1 H	.00736	.00678	.195	.229
.2 H	.01065	.01005	.098	.125
.3 H	.01165	.01124	.037	.056
.4 H	.01134	.01119	-.0006	.0116
.5 H	.01033	.01042	-.0246	-.0179
.6 H	.00898	.00929	-.0384	-.0356
.7 H	.00754	.00803	-.0437	-.0432
.8 H	.00618	.00681	-.0407	-.0410
.9 H	.00502	.00569	-.0285	-.0284
1.0 H	.00372	.00426	-.0052	-.0042

of the walls, slightly underestimates the interaction moments on the frame when the walls are stiffer. In the case of slender wall, the interaction moments on the frame and the moments on the walls are overestimated in the lower storeys while the frame shear forces are overestimated in the upper storeys.

5.7 LIMITING STIFFNESS RATIO FOR THE SHEAR WALL

In general, as the stiffness of the shear wall is increased relative to the interconnected frame, the share of the applied load carried by the frame is reduced. However, it is expected that beyond certain limiting value of the stiffness ratio of the wall, any further increase in the wall stiffness may not significantly reduce the forces on the frame. To get an idea of the limiting ratio of shear wall stiffness for a given frame wall system, the interaction moments on the frame at two typical floor levels and the frame shears at three different storeys are plotted in Figs. 5.10 and 5.11 for a two bay 20 storey frame interconnected with a shear wall (Fig. 5.2). The stiffness of the frame is represented by the ratio $\alpha_1 = \frac{I_c}{I_b} \frac{L_b}{L_c}$, where I_c, L_c, and I_b, L_b refer to the moment of inertia and span for the column

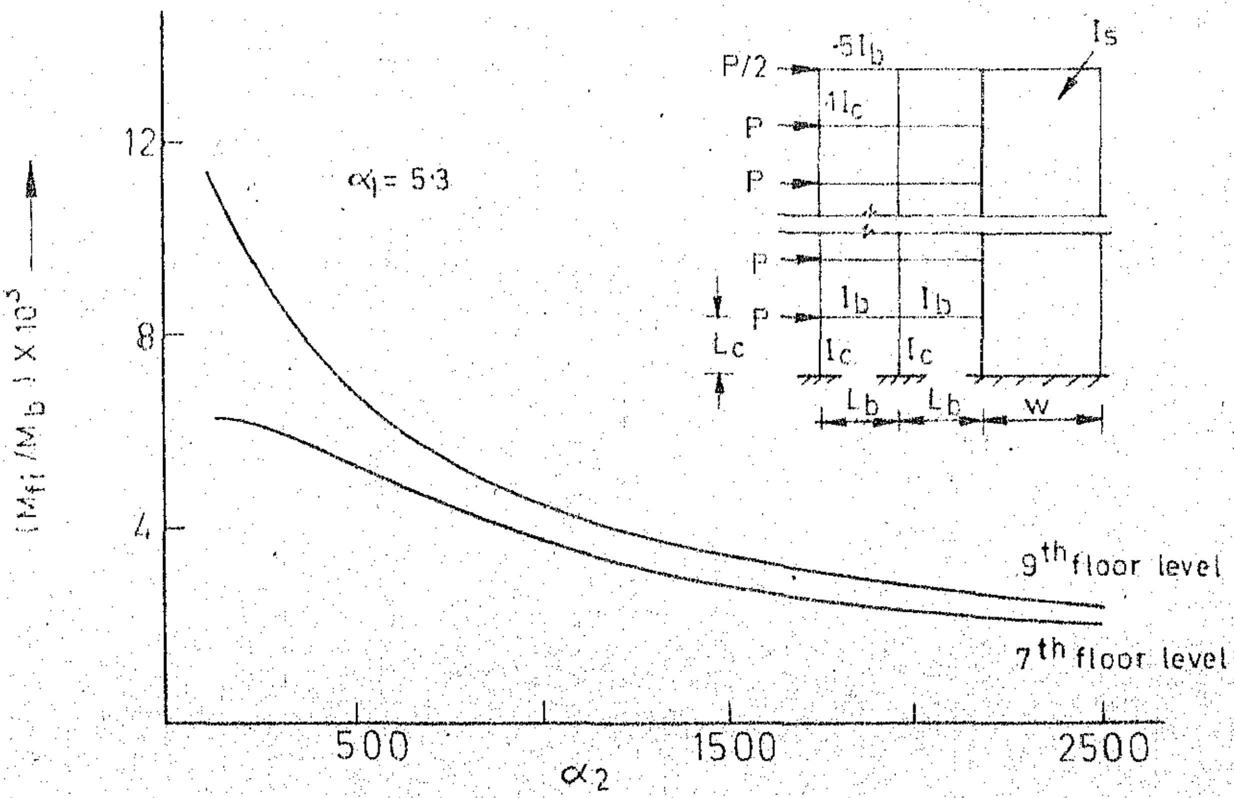


FIG. 5.10 INTERACTION MOMENTS ON FRAME IN 20 STOREY SHEAR WALL FRAME STRUCTURE

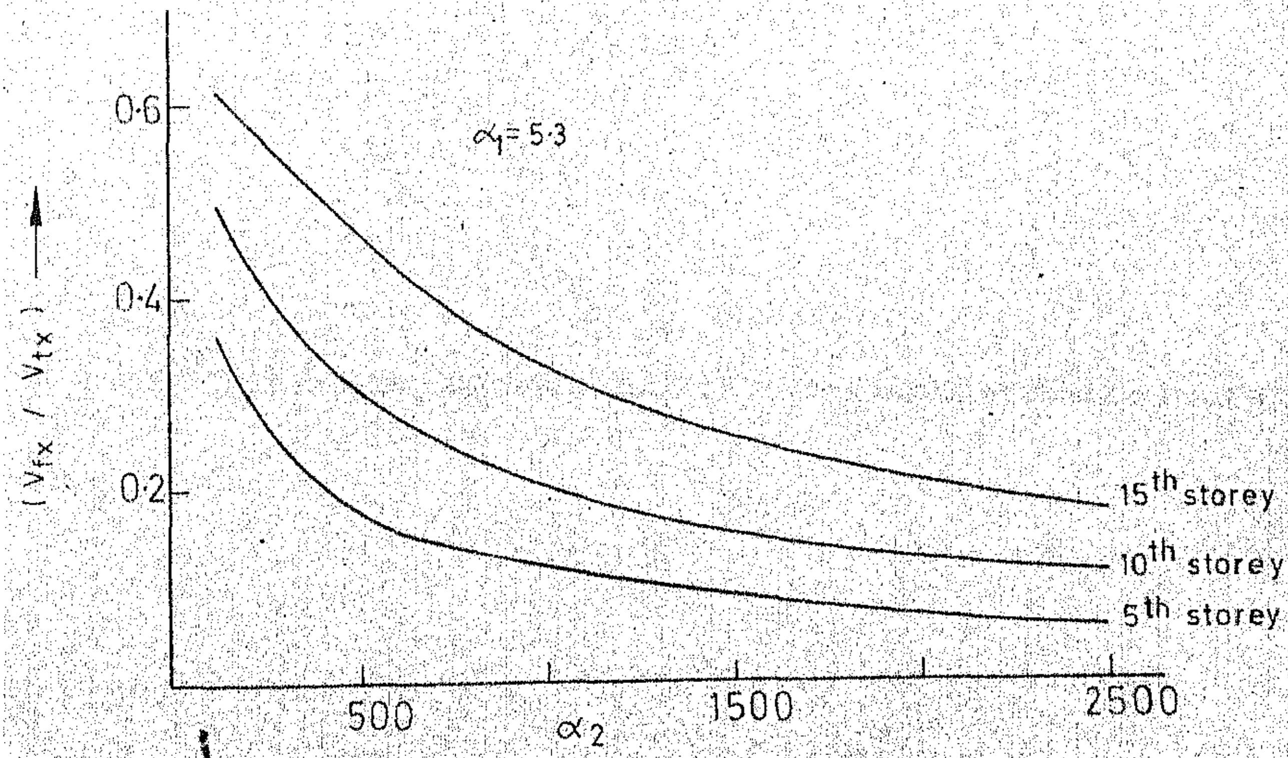


FIG. 5.11 SHEAR FORCES IN FRAME (20 STOREY SHEAR WALL FRAME STRUCTURE)

and beam members in the first storey. The relative stiffness of the shear wall is represented by parameter

$$\alpha_2 = \frac{I_s}{I_b} \frac{L_b}{L_c} \times \frac{(10)^2}{(n_s)^2}$$

where I_s refers to the moment of inertia of the wall and n_s refers to the total number of storeys.

The values of α_2 considered herein vary from 125 to 2500. In Fig. 5.10, the ordinate refers to the interaction moments on the frame at the 4th and 8th storeys of the frame as a fraction of the moments (M_b) at the base due to the applied loads. It can be observed that for α_2 greater than 1500, the interaction moments on the frame do not decrease appreciably. Fig. 5.11 gives the variation of shear carried by the frame in three typical storeys for different stiffness ratios of shear wall. As in the case for interaction moments, it can be seen that for values of α_2 greater than 1500, the reduction in the storey shears of the frame is not substantial. The same trend has been noticed for some other 20 storey structures with different stiffnesses for the frame. From this study, it may be stated that for the 20 storey frame considered, 1500 is the limiting value of shear wall stiffness ratio (α_2) for an efficient participation of the frame in resisting the lateral loads.

The interaction analysis using the iteration scheme for shear wall-frame systems could be applied with equal convenience for the analysis of the structural system in which the shear wall lies between two frames co nected at floor levels (*Fig.* 5.12). As discussed in Chapter 2, the total lateral loads are applied on the shear wall and the free displacements of the walls are computed. An initial deflected shape is assumed for the wall and the frames on either side are subjected to the same set of displacements as of the respective edges of the wall connected to the frames. The interaction forces on each of the frames required to subject the frames to the above displacements are evaluated separately. These interaction forces are reversed in sense, applied on the shear wall and the displacements are evaluated. The convergence of the algebraic sum of the displacements of the wall caused by the interaction forces and the free displacements of the wall with the initial assumed displacements is checked. The iterations are repeated till the specified convergence criterion is satisfied.

Also when the frames and walls do not lie in the same plane but are symmetrically distributed, interconnected to each other through floor diaphragms, all the frames

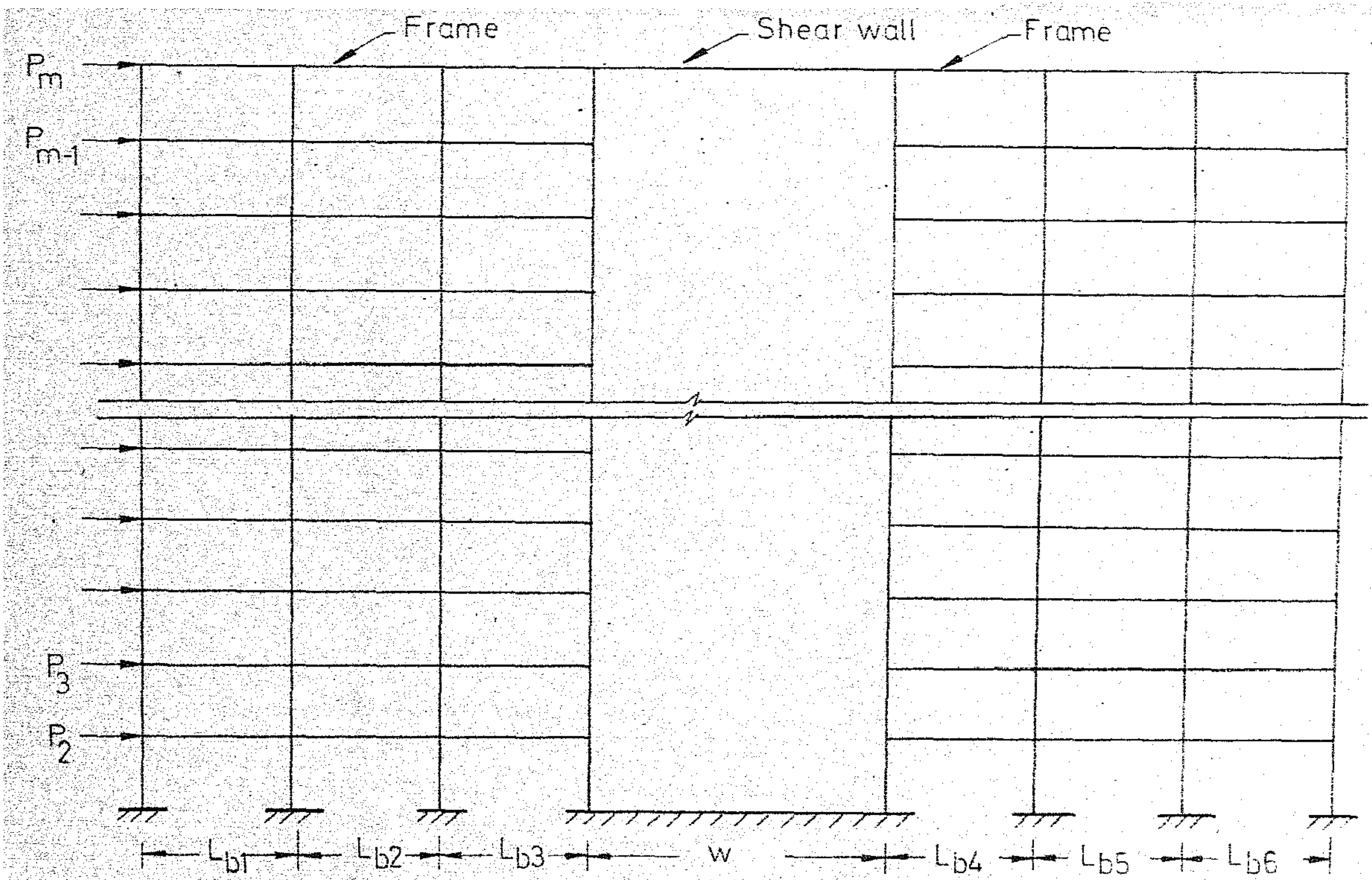


FIG. 5.12 SHEAR WALL WITH INTERCONNECTED FRAMES

could be replaced by simple frame of equivalent stiffness. The group of shear walls in the original system may be replaced by a single shear wall of equivalent stiffness and width. The interaction analysis is carried out on the replaced structure consisting of a single equivalent frame and the interconnected shear wall. The forces in the columns, beams and the shear wall of equivalent system are evaluated and the corresponding forces in the individual columns, beams and walls of the original system are obtained by distributing the forces in the replaced system in proportion to their relative stiffnesses.

5.8 DESIGN CURVES

The design of shear wall and interconnected frame requires the evaluation of the forces of interaction as a first step. Next, the forces in the members of the frame are computed by any convenient method of frame analysis and the shear forces and moments at different levels on the shear wall are obtained by statics. The evaluation of the interaction forces requires large amount of computations and this increases in proportion with the increase in the number storeys of the structure. To facilitate the design of frame and shear wall structures, the interaction forces due to applied lateral loads, have been evaluated in terms of non-dimensional coefficients and are presented in the form of charts for a wide range of structural proportions of shear wall and frame. Separate charts have been provided for shear walls with openings and for solid shear walls.

A 20 storey shear wall and frame system, as shown in Fig. 5.13, has been used for the preparation of charts giving the coefficients of design forces on the shear wall and frame. The storey heights are equal in each of the storeys. The moment of inertia of the columns decreases uniformly from first storey, with its value in the top storey equal to one-tenth of that in the first storey (Fig. 5.5a). The moment of inertia of

the beams also decreases from bottom to top with its value in the top floor equal to one half of that in the first storey (Fig. 5.6a). The shear wall has constant cross-section along the height. The loads are applied at the floor levels.

Coefficients for

(i) shear force (V_{fx}) in each storey of the frame,

(ii) interaction moments (M_f) on the frame (at the points of connection with the wall) and

(iii) moments (M_w) on the wall at different levels along the height

are presented through graphs. The shear force in different storeys of the frame can be computed from

$$V_{fx} = C_s V_{tx} \tag{5.3}$$

where 'C_s' is the coefficient available from the graph and V_{tx} refers to the total applied shear in the storey. The coefficients 'C_s' are taken from the graphs corresponding to the mid-height level of the storey under consideration. The interaction moments on the frame are obtained as

$$M_f = C_{im} M_b \times \left(\frac{n_s}{20}\right) \tag{5.4}$$

where 'C_{im}' is the coefficient available from the charts, M_b refers to the total moment at the base of the structure caused by the applied loads and ($\frac{n_s}{20}$) is a scale factor determined by the total number of storeys 'n_s'. The moments on the shear wall are evaluated as

$$M_w = C_{wm} M_b \tag{5.5}$$

where C_{wm} is the coefficient available from the charts. Charts have not been provided for the vertical interaction forces, as they do not influence the design of the frame in any significant manner. However, the contribution of the vertical interaction forces to the moments induced in the wall has been considered in the preparation of the charts giving the coefficients 'C_{wm}' of the moments on the wall.

Four sets of charts are presented. Each set deals with one frame stiffness and six different relative stiffness ratios of the shear wall to the frame. The stiffness of the frame is defined by the factor,

$$\beta_1 = \frac{A_b L_b^2}{I_b} \times \frac{I_c}{A_c L_c^2} \tag{5.6}$$

where the spans (L_b, L_c), areas (A_b, A_c) and the moments of inertia (I_b, I_c) of the beam and the column correspond to the first storey. The values of the frame stiffness factor β_1 considered are,

(i) 5

(ii) 10

(iii) 15

(iv) 20

The parameter 'β_2' representing the relative stiffness of the shear wall to the frame is defined as,

$$\beta_2 = \frac{I_s}{\Sigma I_b} \times \frac{L_b}{h} \times \frac{40}{(n_s)^2} \qquad (5.7)$$

The values of the relative shear wall stiffness ratios β_2 used in the preparation of the charts are

(i) 25

(ii) 50

(iii) 100

(iv) 200

(v) 300

(vi) 500

The above values of the frame stiffness factors β_1 and the relative shear wall stiffness ratios β_2, cover a wide range of structural proportions in shear wall and frame structures.

5.8.1 FRAME AND INTERCONNECTED SOLID SHEAR WALL

Figs. 5.14 give the frame shear coefficients 'C_s' for a frame interconnected with a solid shear wall (Fig. 5.13). The shear in any storey of the frame is obtained by taking the value of the coefficient 'C_s' corresponding to the mid-height level of the storey from the appropriate curve and using Eq. 5.3. In most cases, the shear in the top storey of the frame corresponding to the level 0.975 H (for the 20 storey frame considered) shoots upto very high values. When these values could not be accommodated in the plot, they are indicated by their numerical values at the end of the curves. Figs. 5.15 give the coefficients C_{im} of the interaction moments on the frame at the points of connection with the shear wall (Eq. 5.4). Figs. 5.16 offer the coefficients C_{wm} for evaluating the moments on the shear wall at different levels along the height (Eq. 5.5). These charts can also be used when the moment of inertia of the shear wall decreases from bottom to top, as it has been indicated in Section 5.5. that moderate reductions in the moment of inertia of the shear wall along the height do not affect the interaction behaviour significantly.

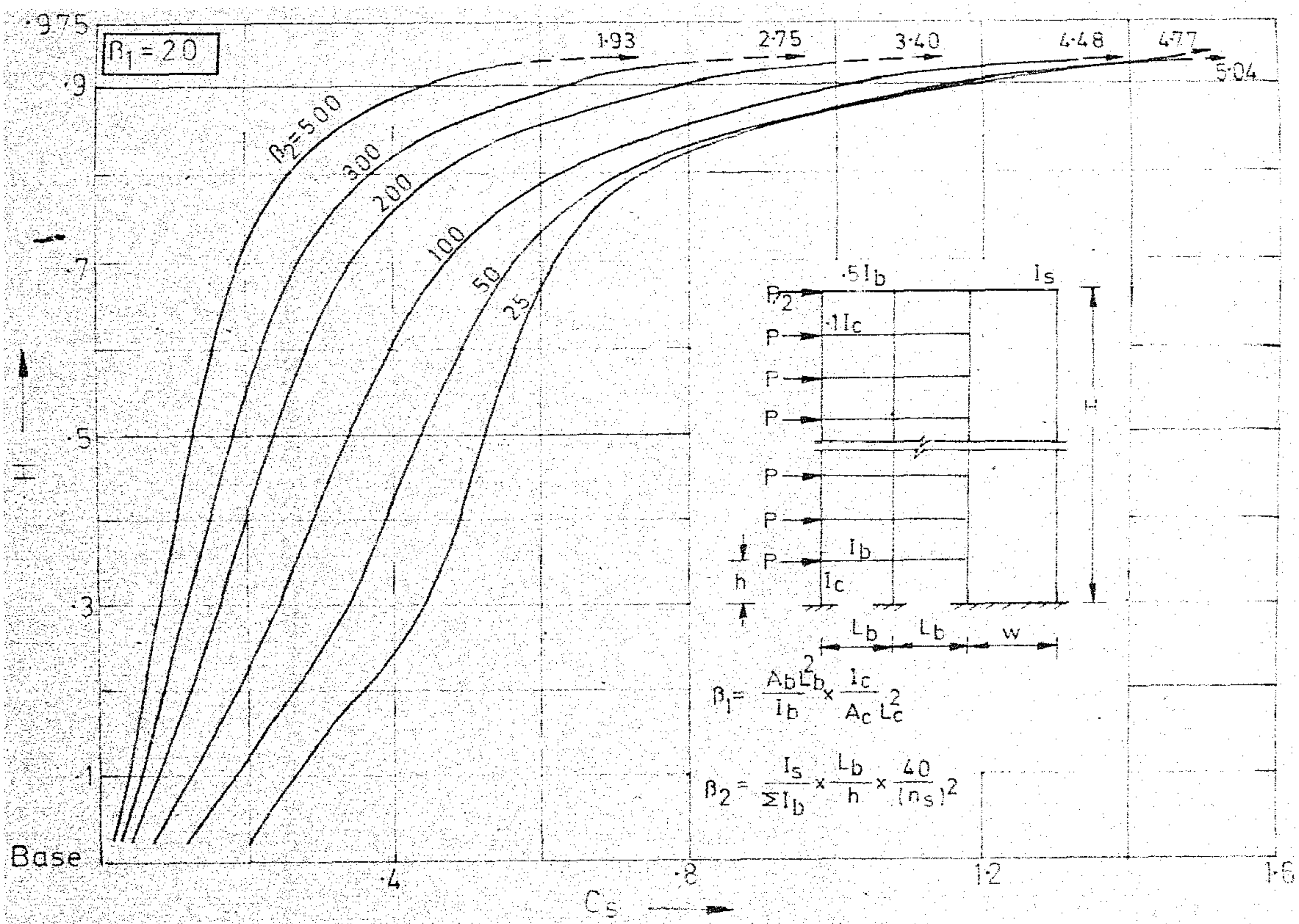


FIG. 5·14 a COEFFICIENTS OF SHEAR FORCES IN FRAME

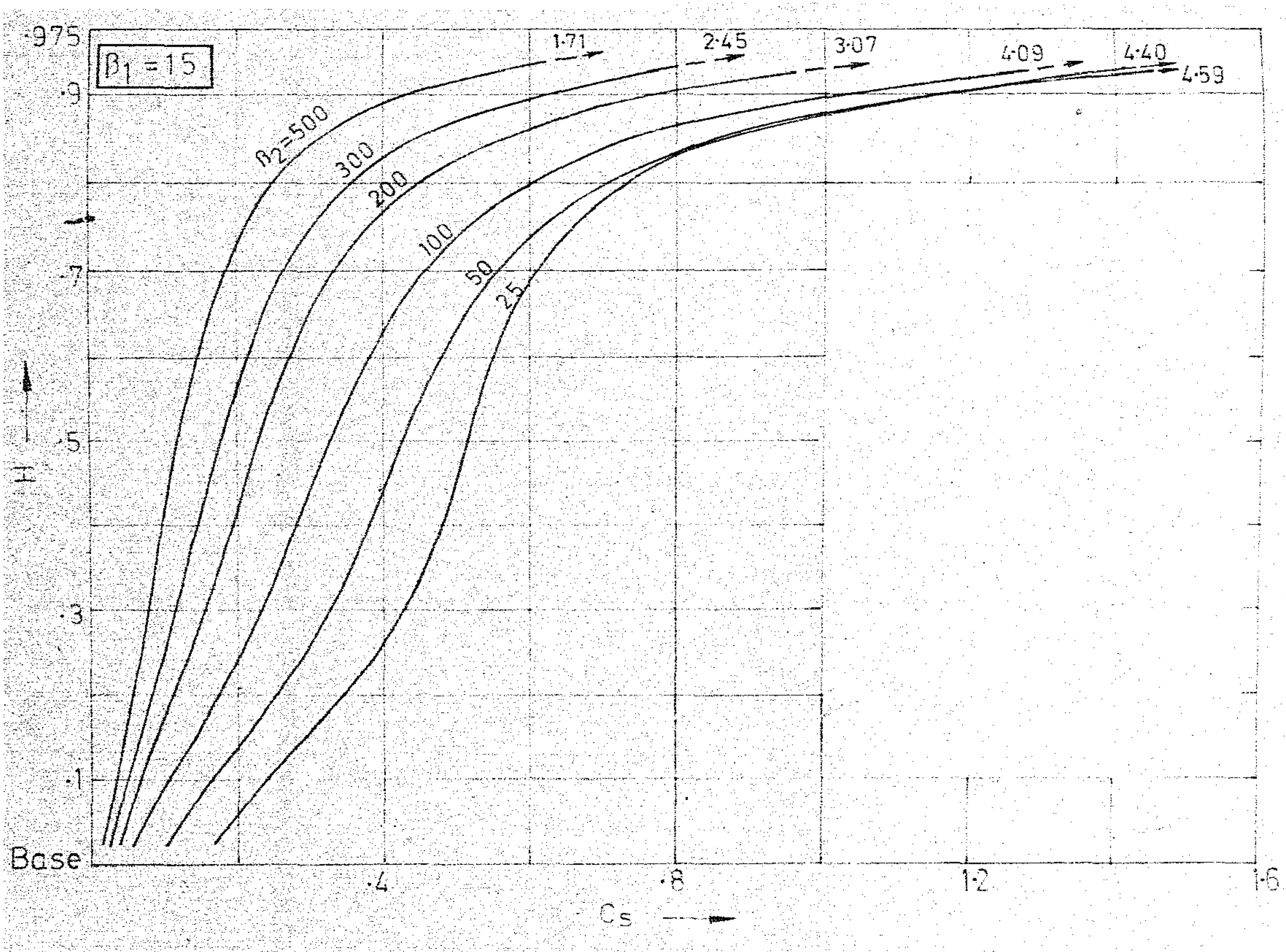


FIG. 5·14 b COEFFICIENTS OF SHEAR FORCES IN FRAME

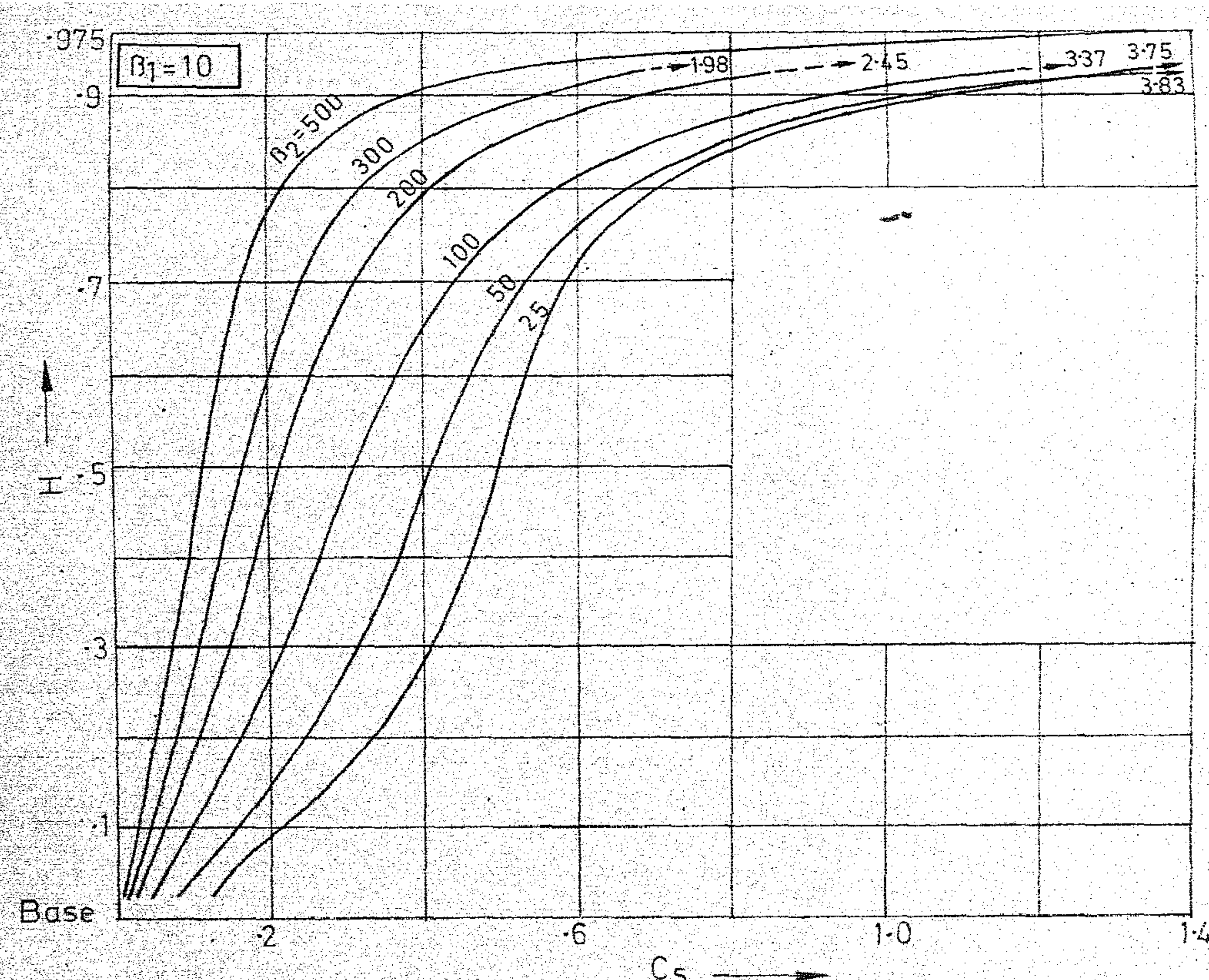


FIG.5·14 c COEFFICIENTS OF SHEAR FORCES IN FRAME

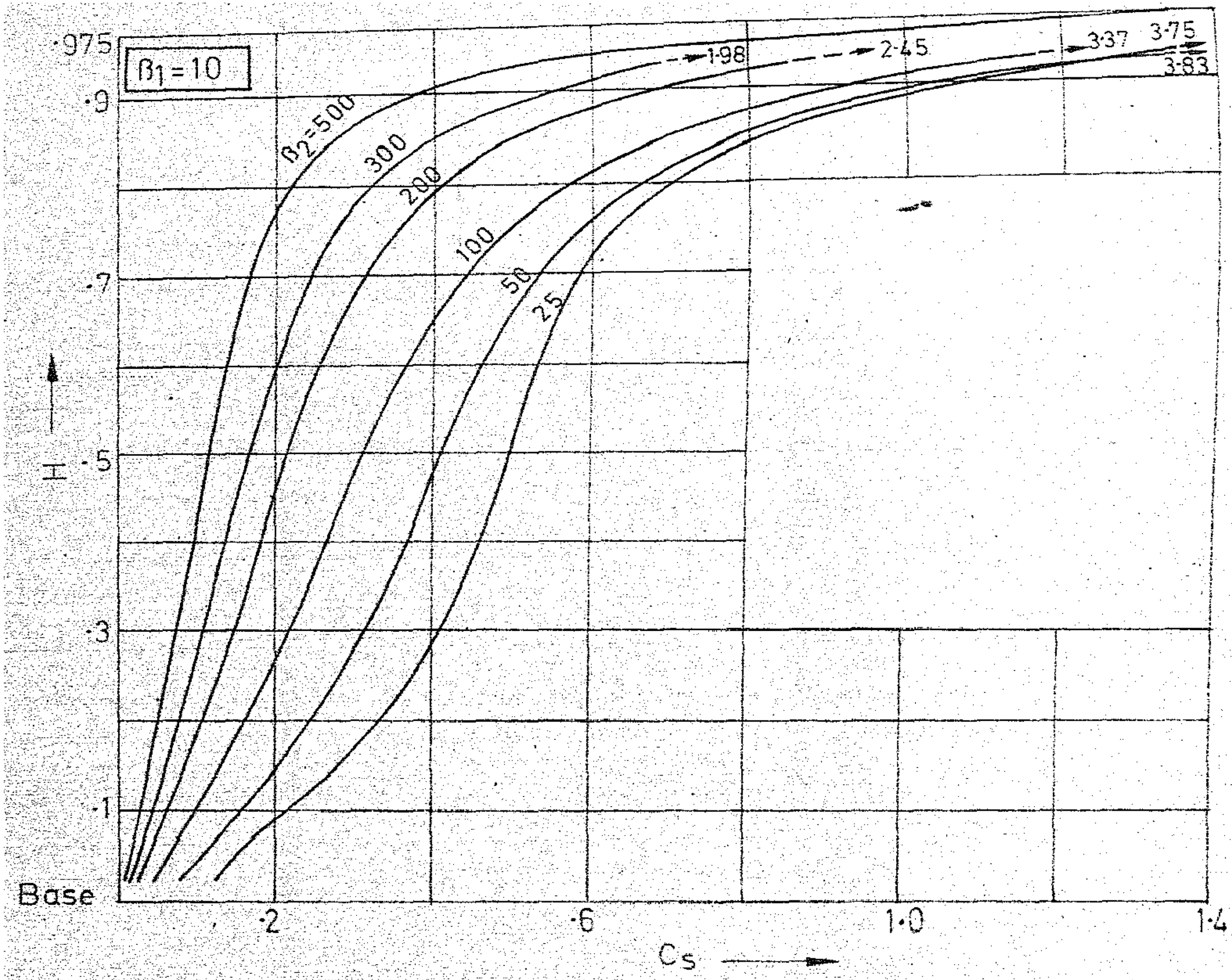


FIG.5·14 c COEFFICIENTS OF SHEAR FORCES IN FRAME

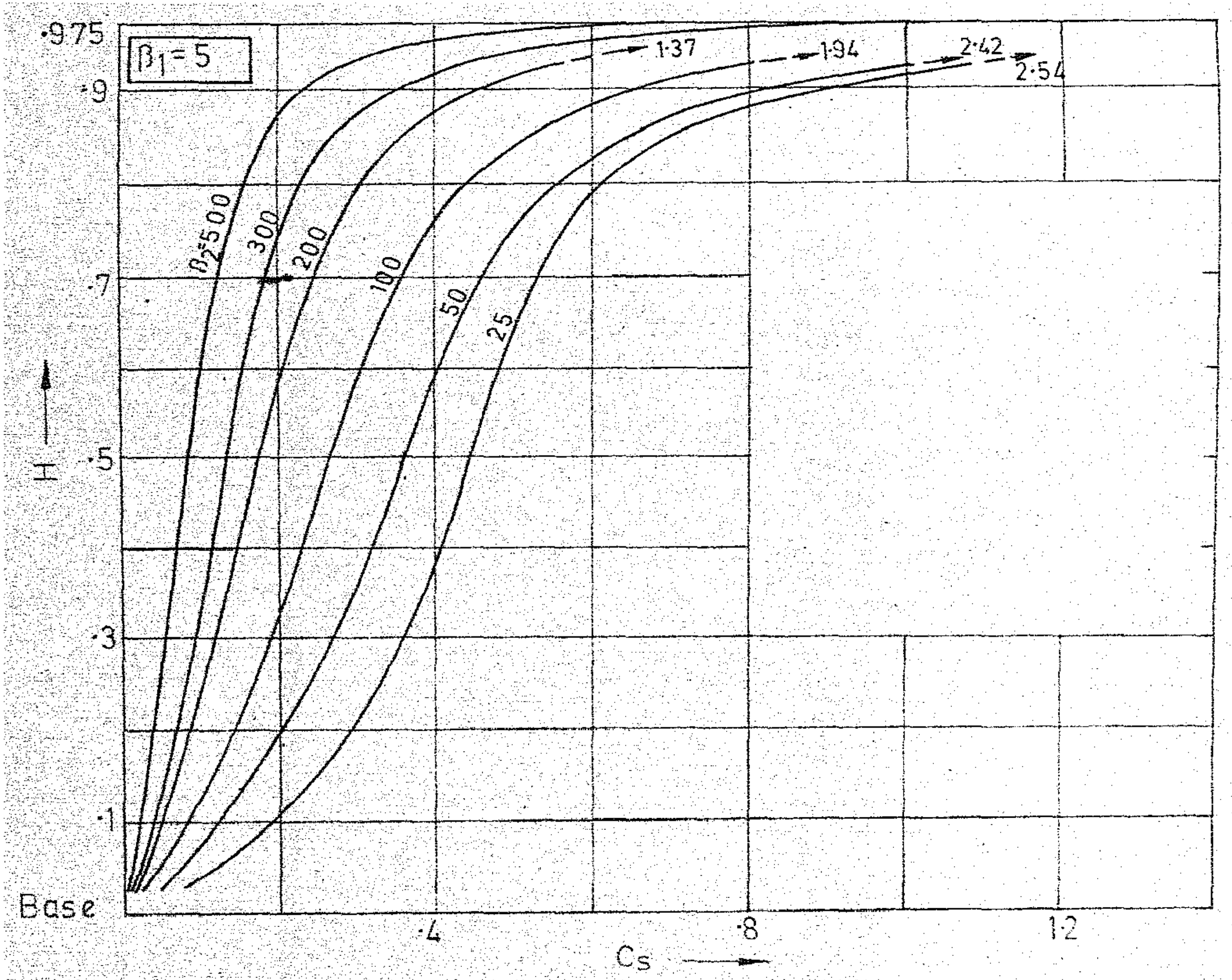


FIG. 5·14 d COEFFICIENTS OF SHEAR FORCES IN FRAME

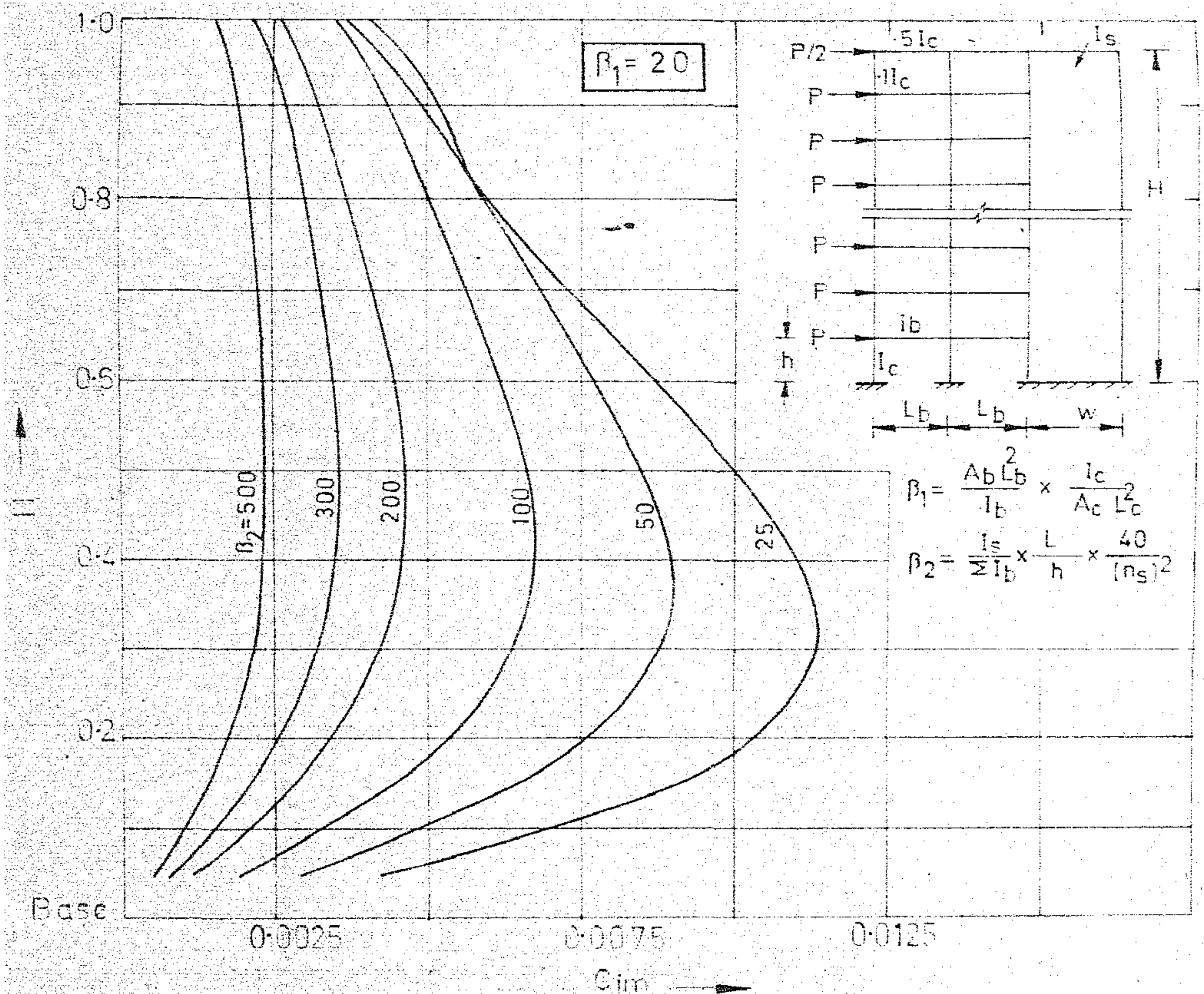


FIG. 5·15 a COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

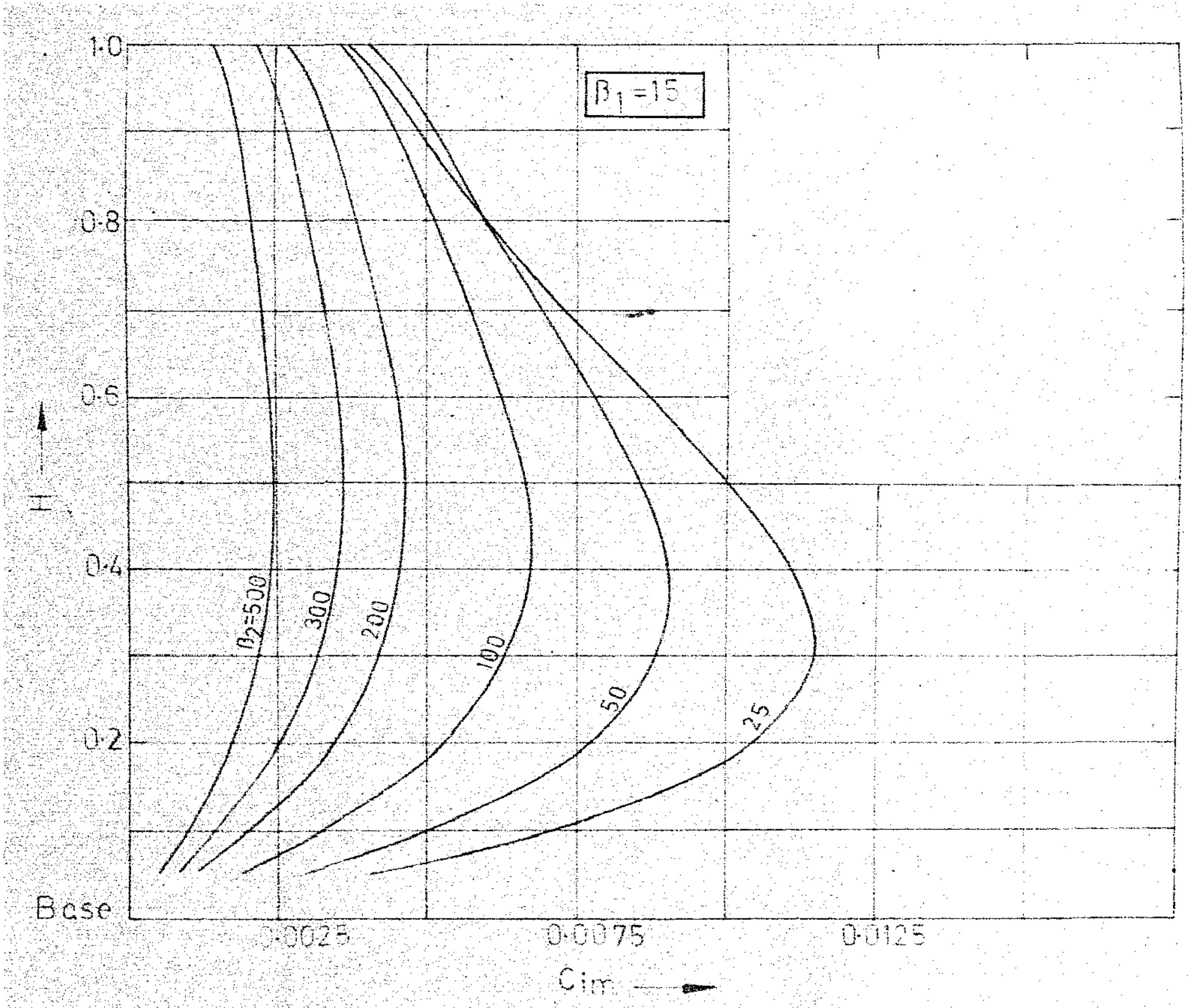


FIG. 5·15 b COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

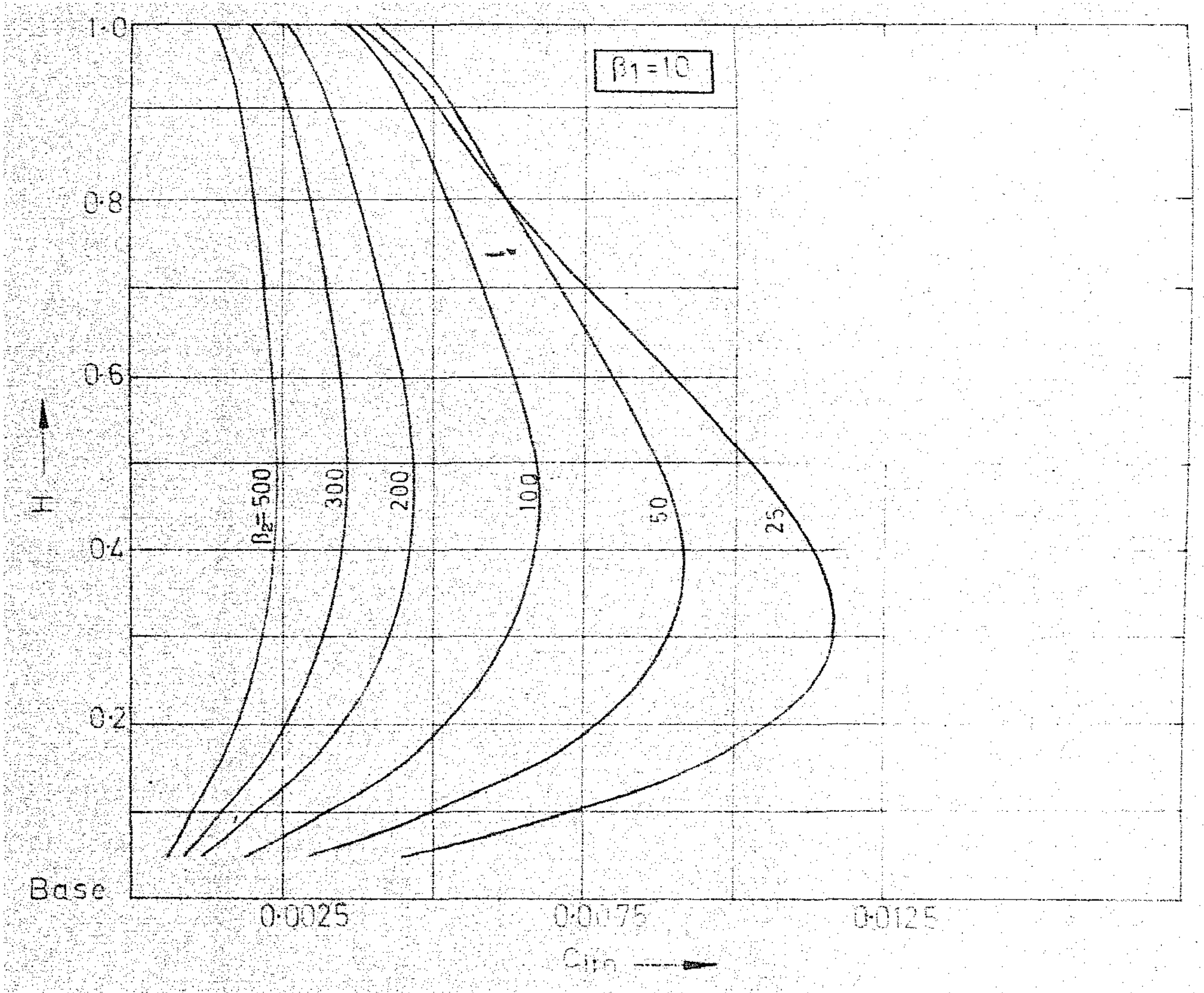


FIG. 5·15 c COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

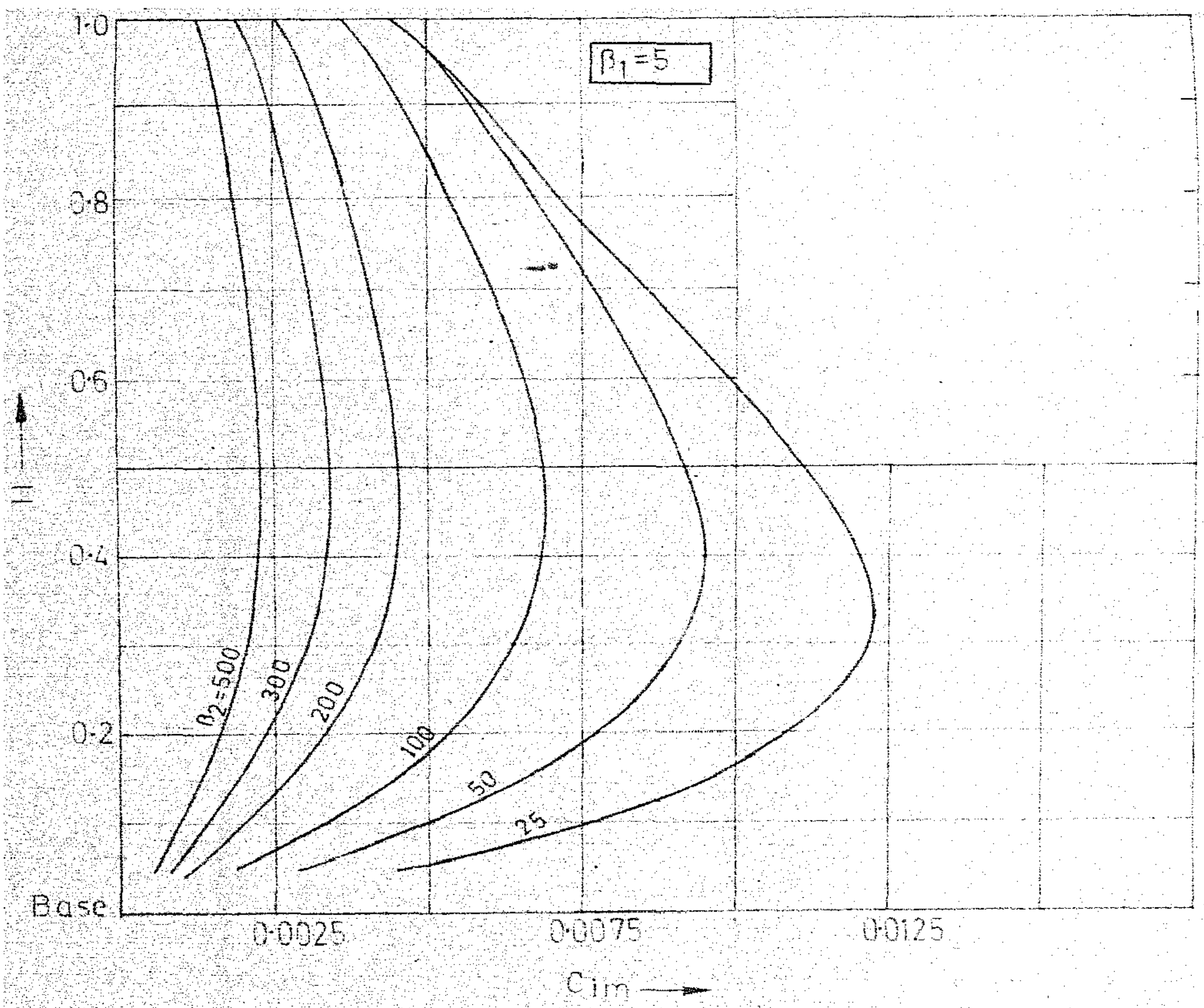
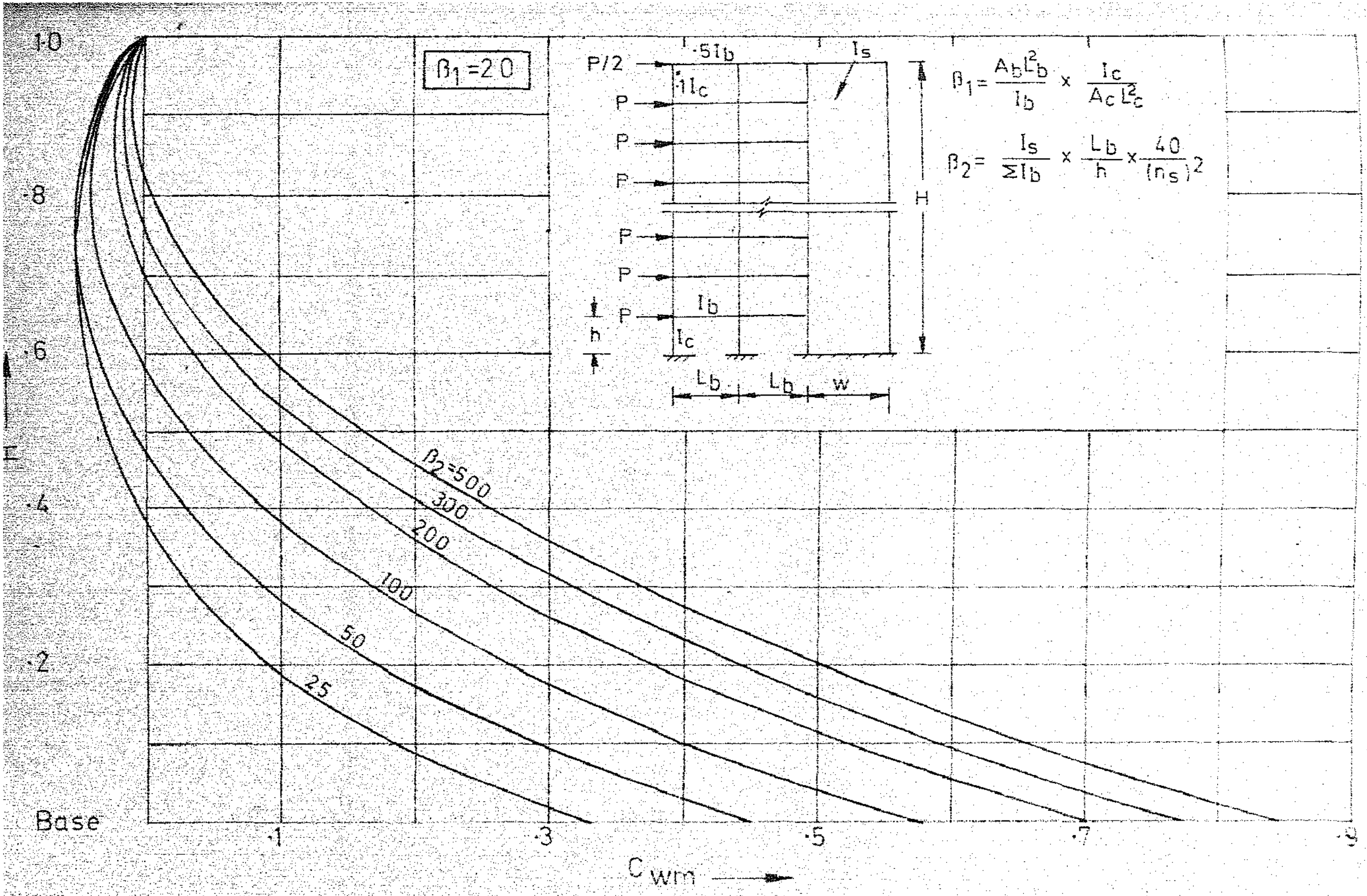


FIG. 5·15 d COEFFICIENTS OF INTERACTION MOMENTS ON FRAME



FIG·5·16 a COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

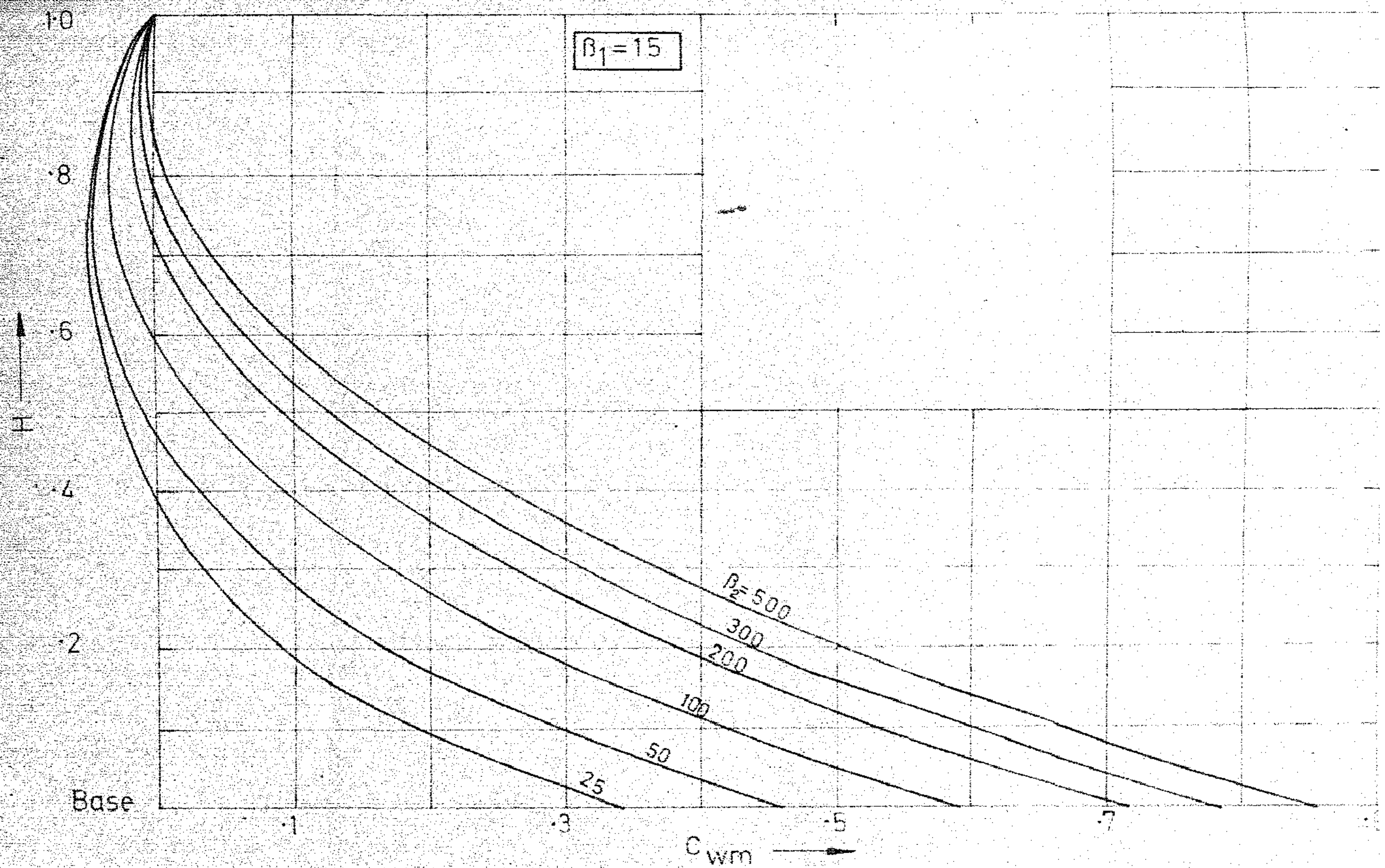


FIG. 5·16b COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

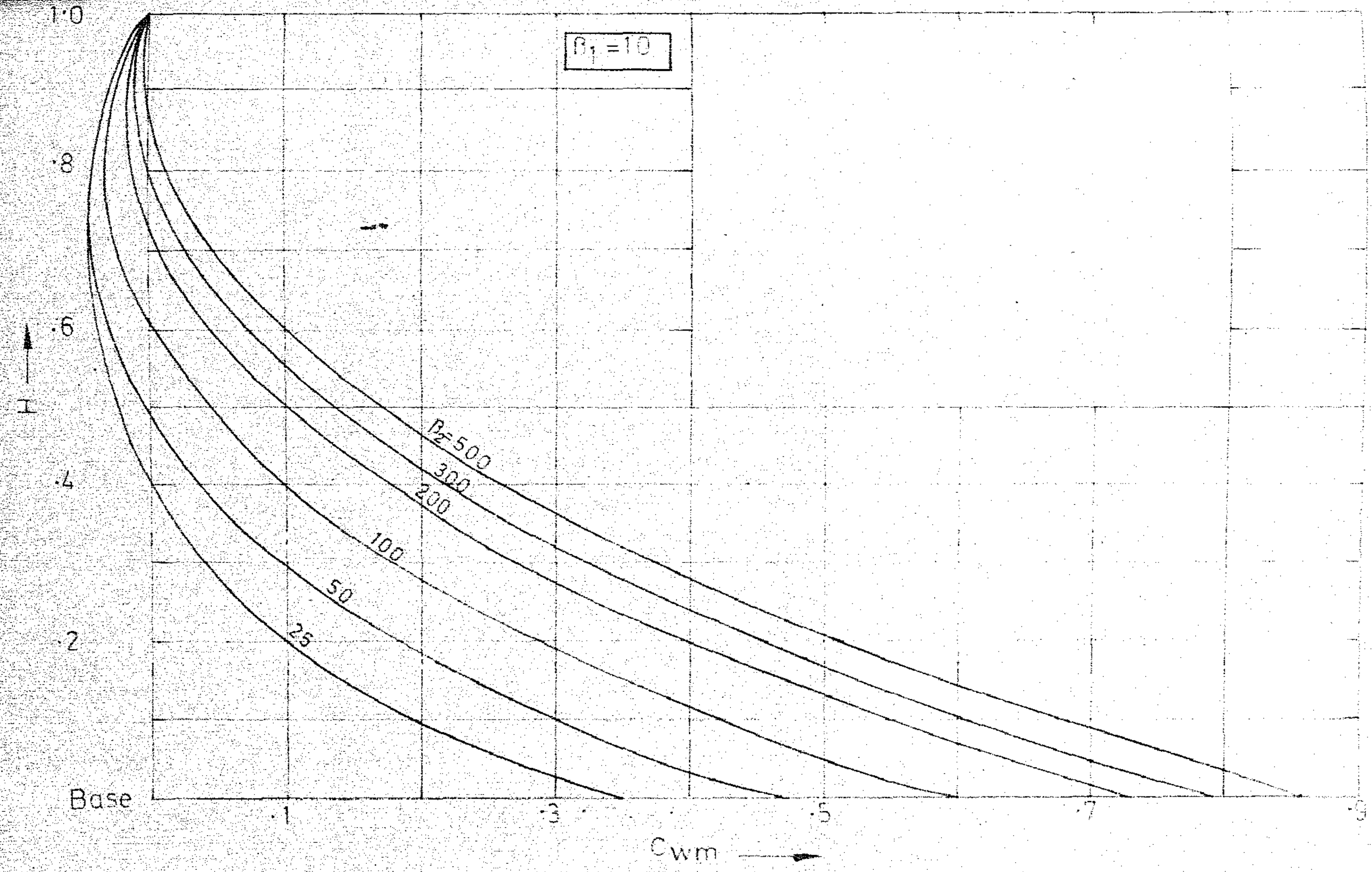


FIG. 5·16 c COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

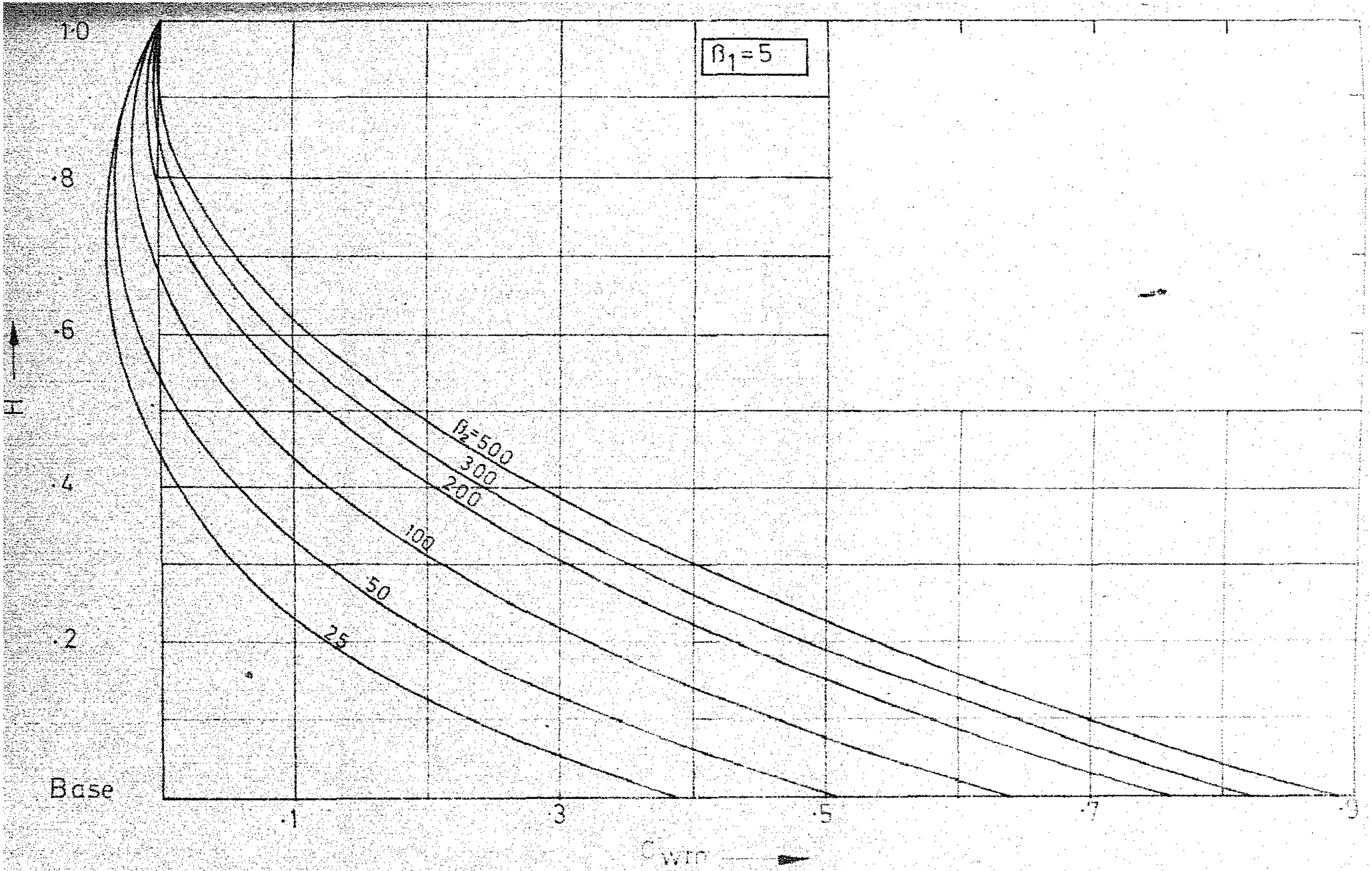


FIG.5·16.d COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

5.8.2 FRAME AND SHEAR WALL WITH OPENINGS

Coefficients for

(i) shear force in each storey of the frame,

(ii) interaction moments on the frame at the points of connection with the shear wall and

(iii) moments on the wall at different levels are presented through charts (Figs. 5.18 to 5.20) for frame interconnected with shear wall having openings (Fig.5.17).

Central rectangular openings of equal size in each of the storeys are considered. The base of the openings lies at floor levels. The width of the opening is equal to one half that of the wall and height is equal to 0.7 times that of the storey height. Thus the opening in each storey covers 35 percent of the area of the wall and corresponds to the limiting size of the openings in the shear wall for which the beam theory could be used to compute the displacements of the wall (discussed in Chapter 3). The shear wall has constant width and thickness along the height. In Fig. 5.17, I_s refers to the moment of inertia of the wall at any section where no opening exists. Figs. 5.18 give the coefficients 'C_s' for the shear forces in the frame (Eq. 5.3).

Figs. 5.19 give the coefficients 'C_{im}' of the interaction moments on the frame at the points of connection with the wall, (Eq. 5.4). Figs. 5.20 give the values of the coefficients 'C_{wm}' of the moments on the wall at different levels along the height (Eq. 5.5).

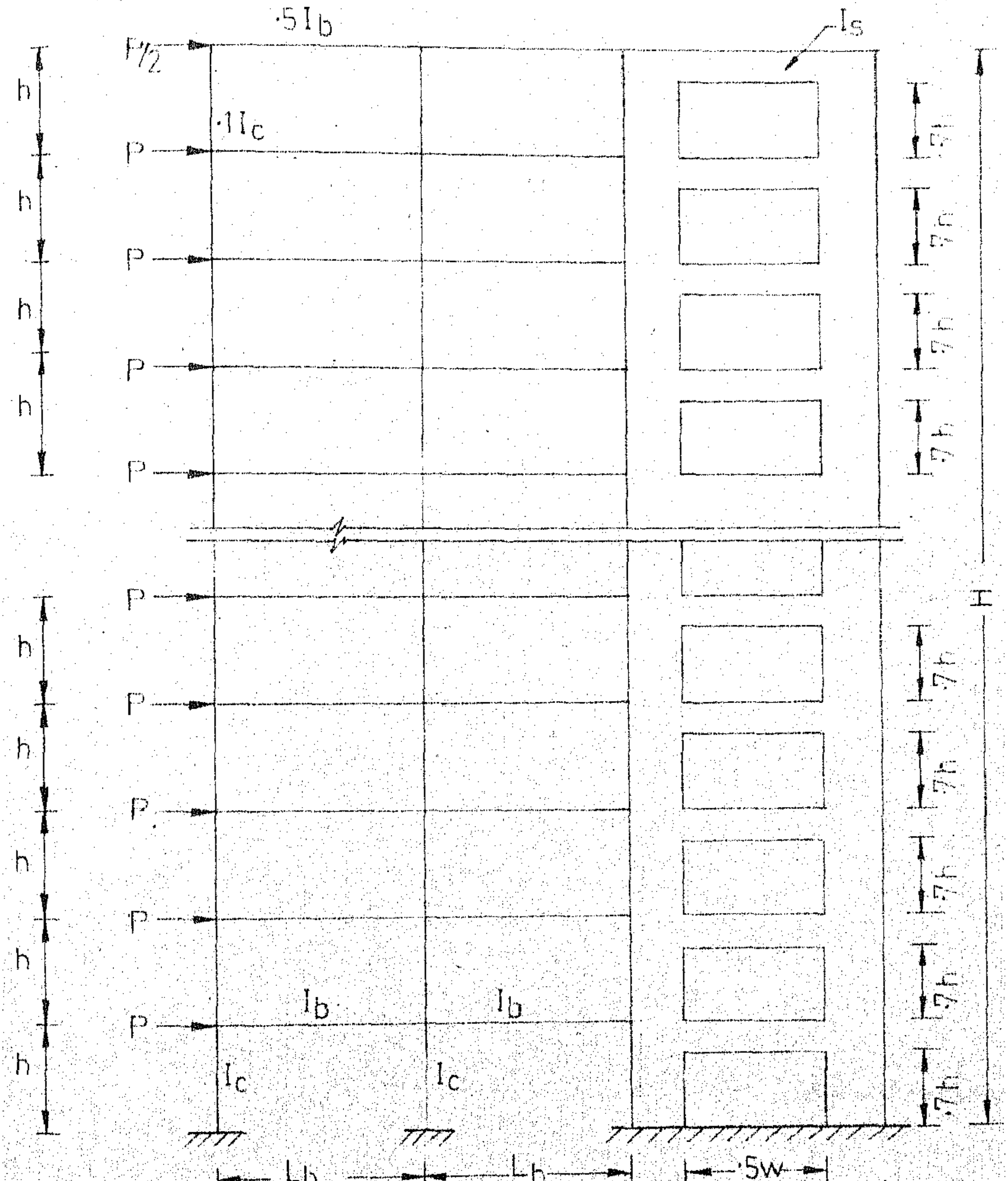
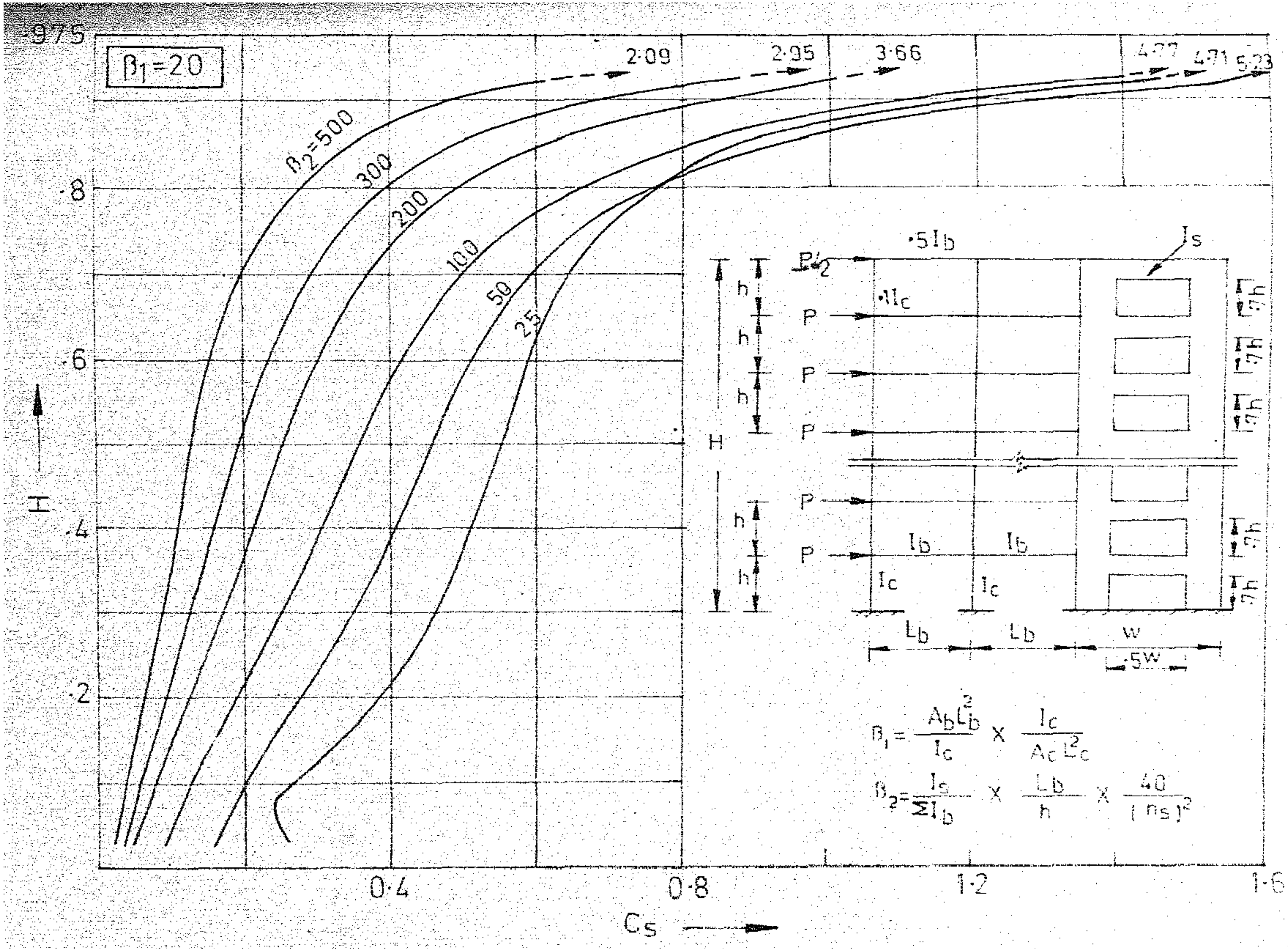


FIG.5·17 20 STOREY FRAME AND INTERCONNECTED SHEAR WALL WITH OPENINGS



$$\beta_1 = \frac{A_b L_b^2}{I_c} \times \frac{I_c}{A_c L_c^2}$$

$$\beta_2 = \frac{I_s}{\Sigma I_b} \times \frac{L_b}{h} \times \frac{40}{(ns)^2}$$

FIG. 5·18 a COEFFICIENTS OF SHEAR FORCES IN FRAME

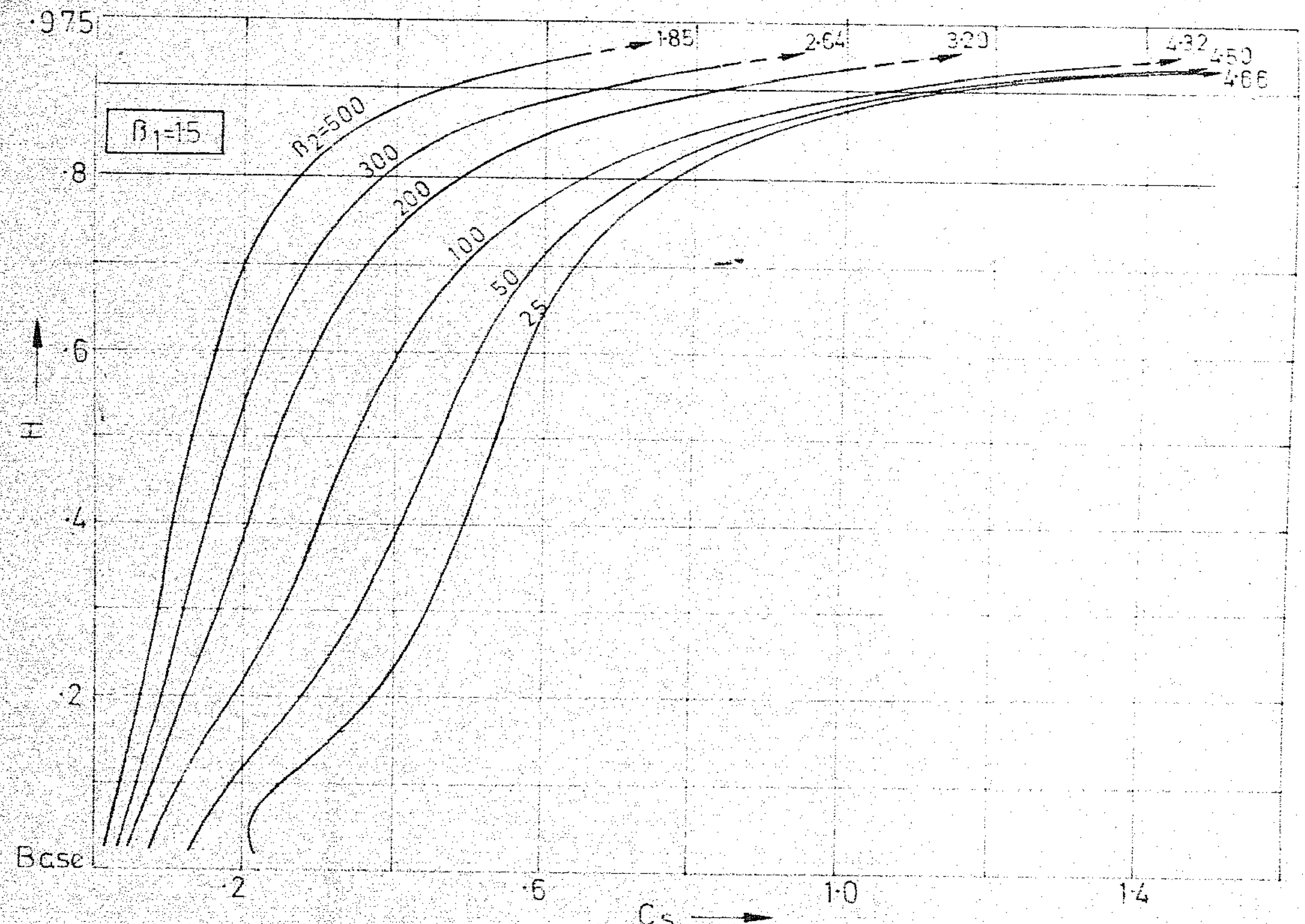


FIG. 5·18 b COEFFICIENTS OF SHEAR FORCES IN FRAME

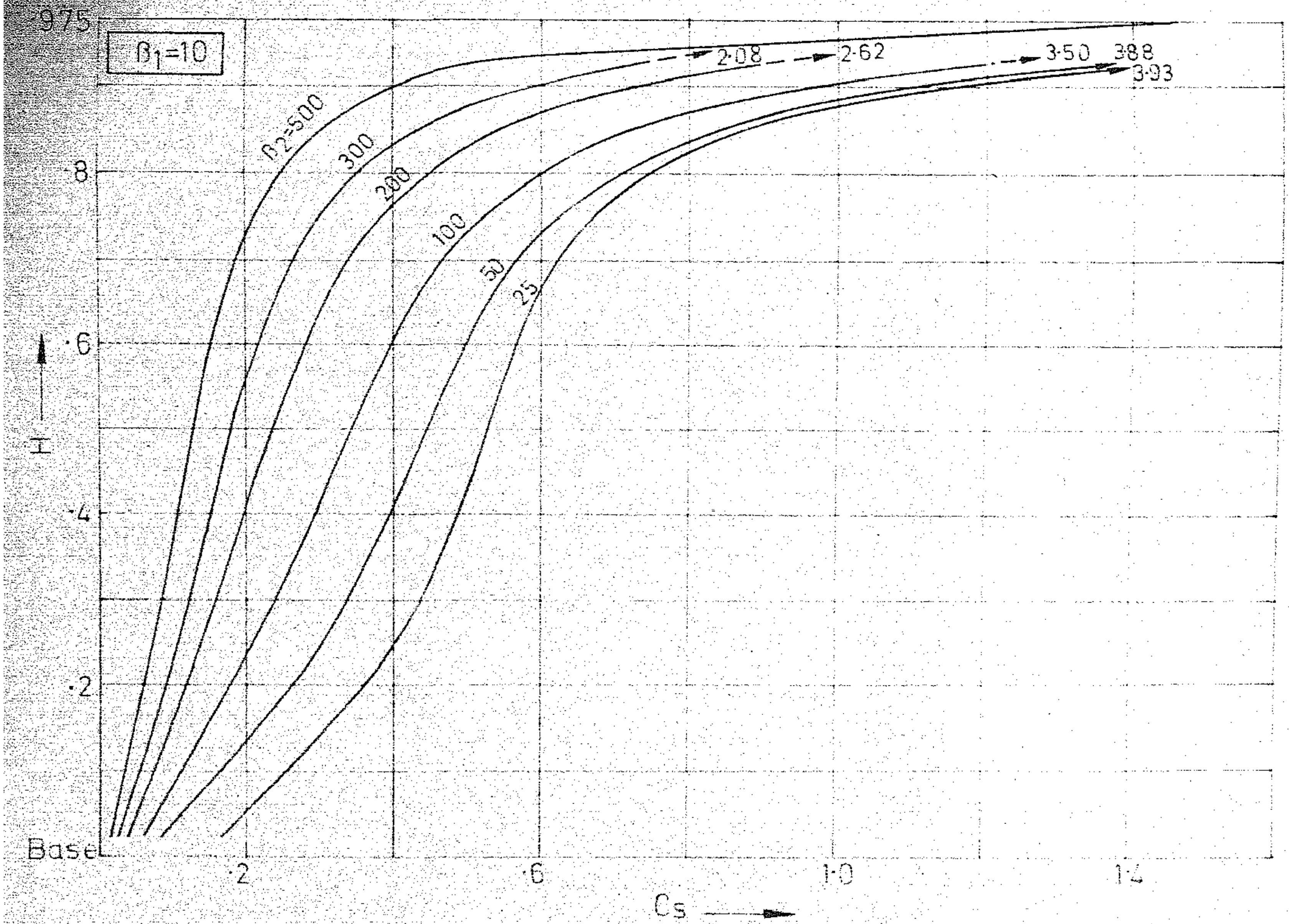
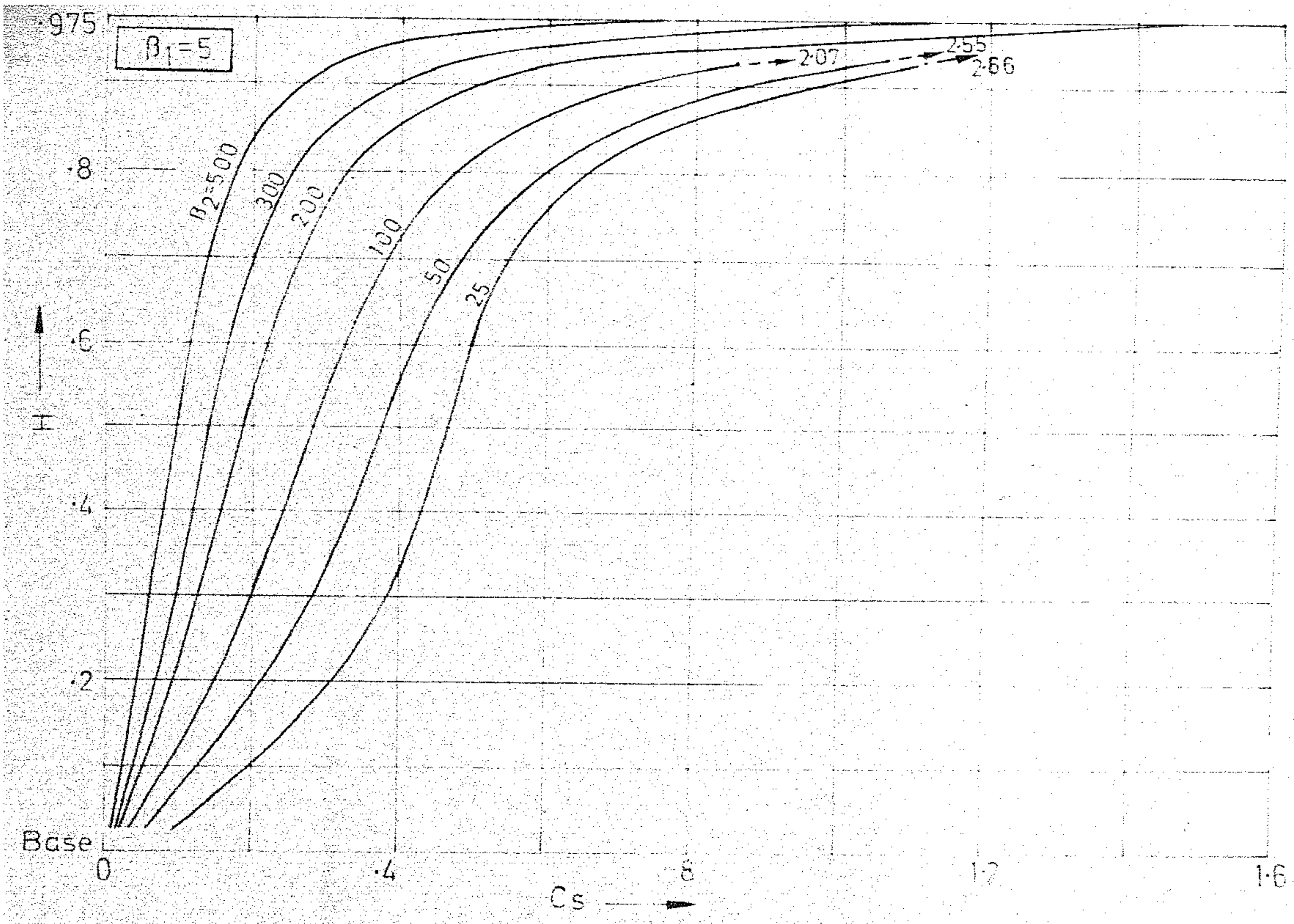
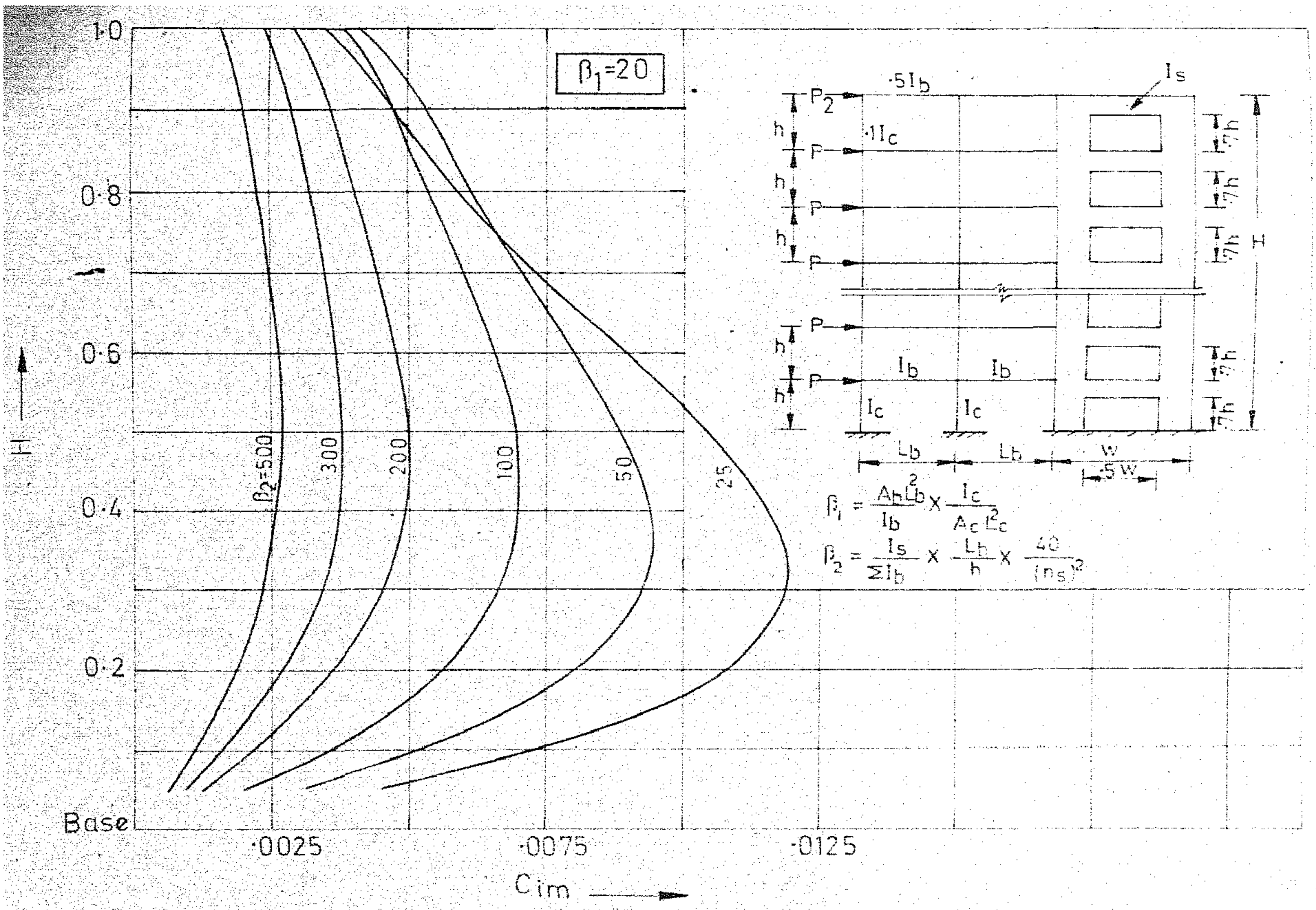


FIG. 5·18 c. COEFFICIENTS OF SHEAR FORCES IN FRAME



FIG·5·18 d COEFFICIENTS OF SHEAR FORCES IN FRAME



FIG·5·19 a COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

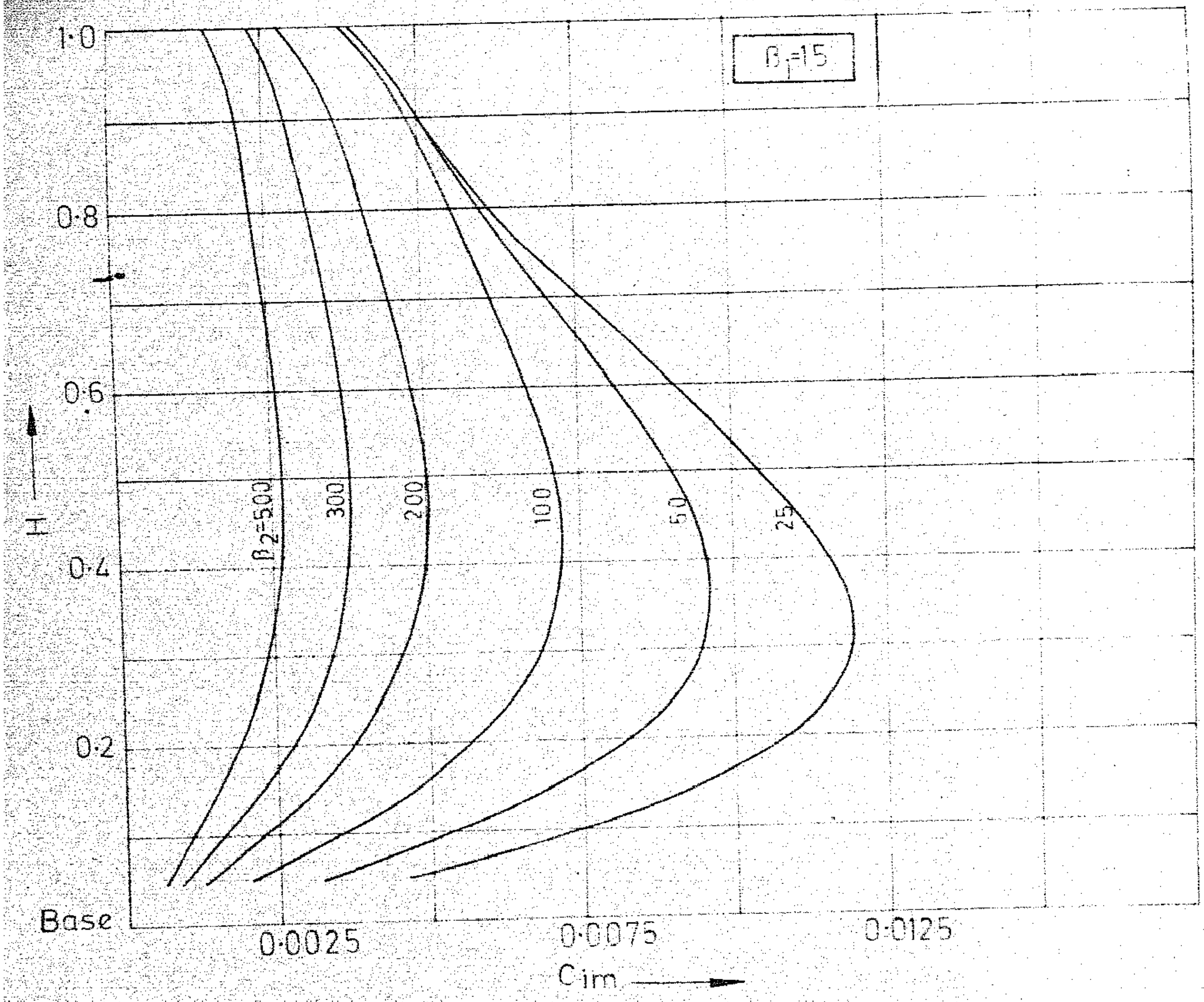


FIG.5·19 b COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

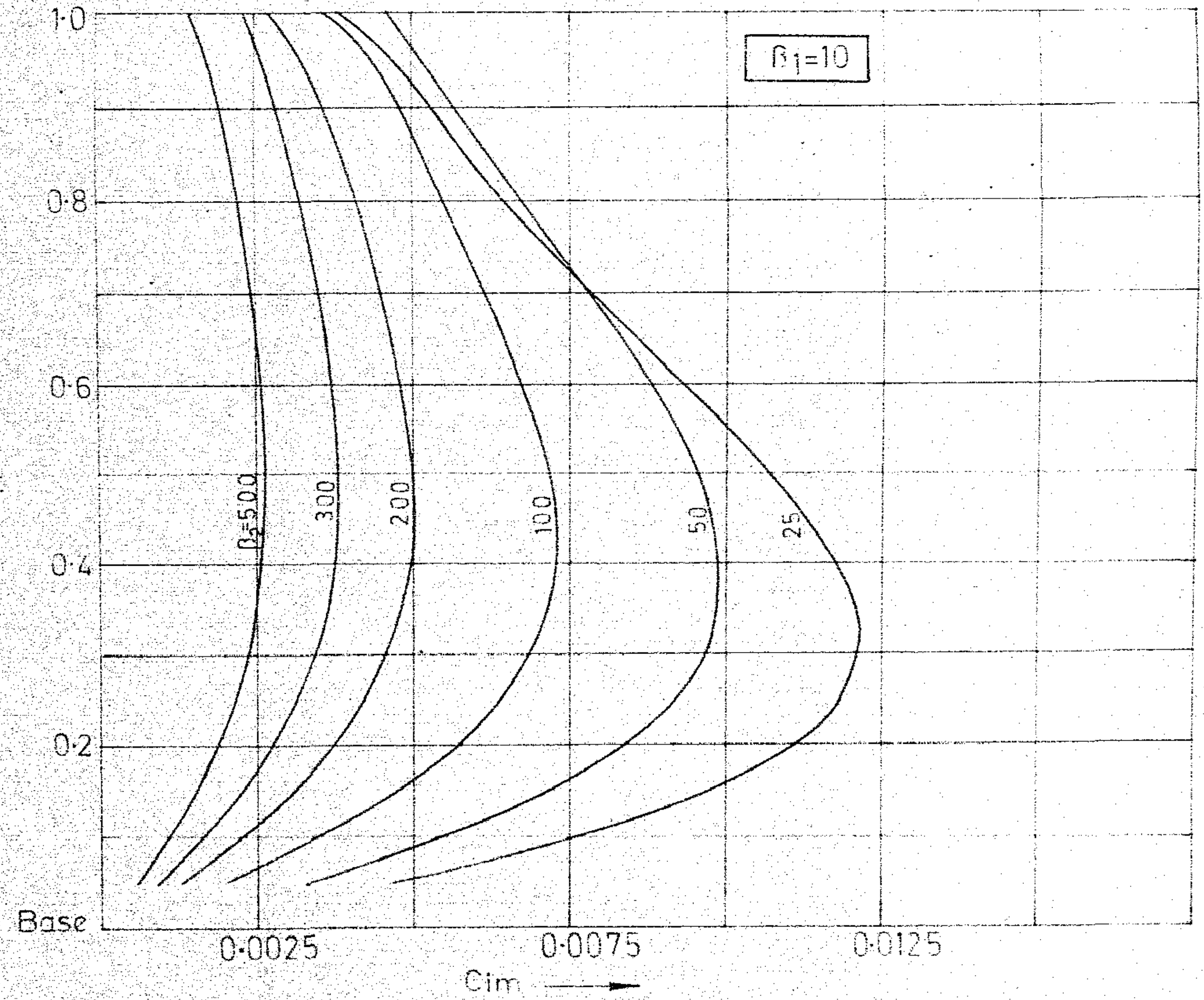


FIG. 5·19 c COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

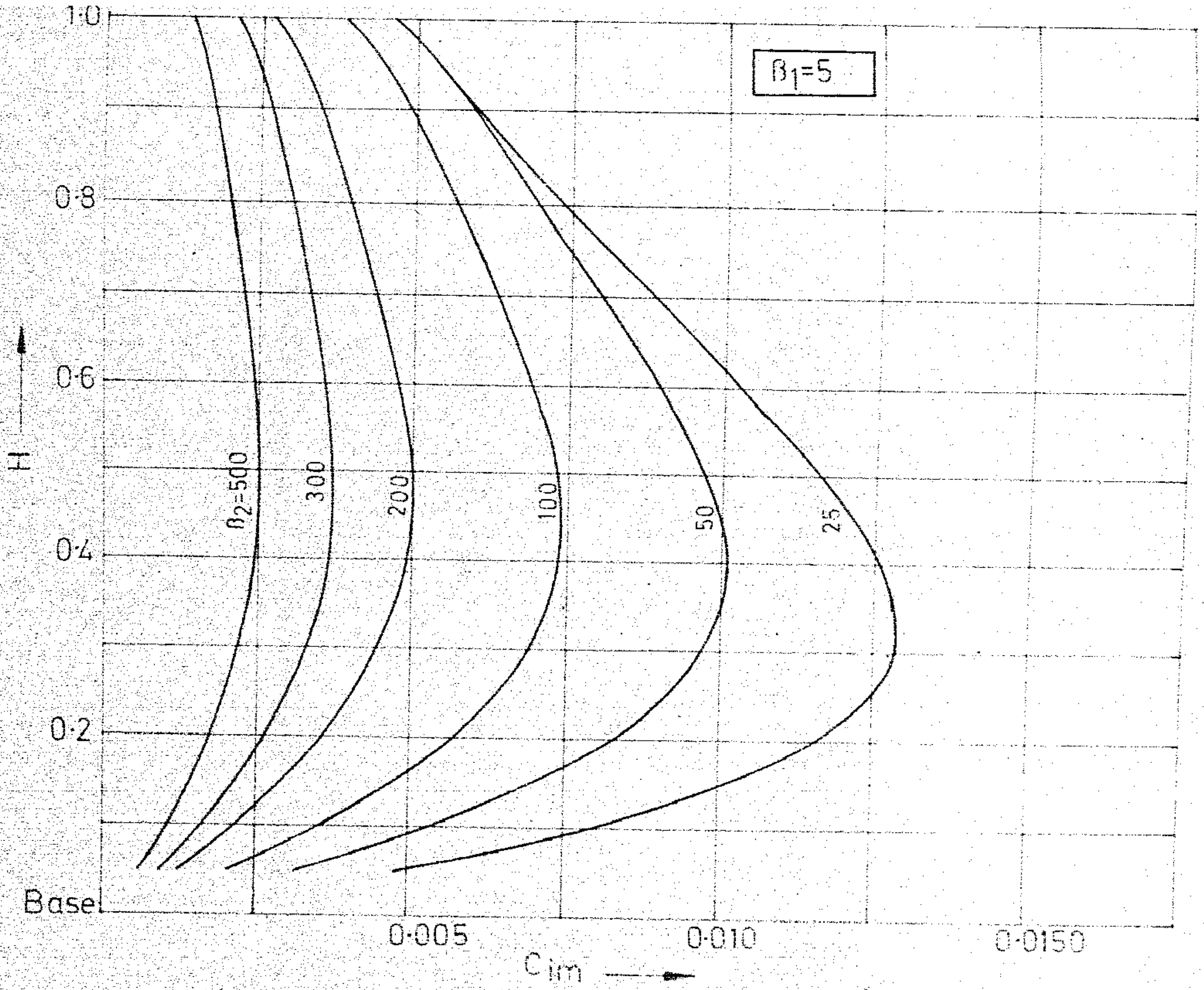


FIG. 5·19 d COEFFICIENTS OF INTERACTION MOMENTS ON FRAME

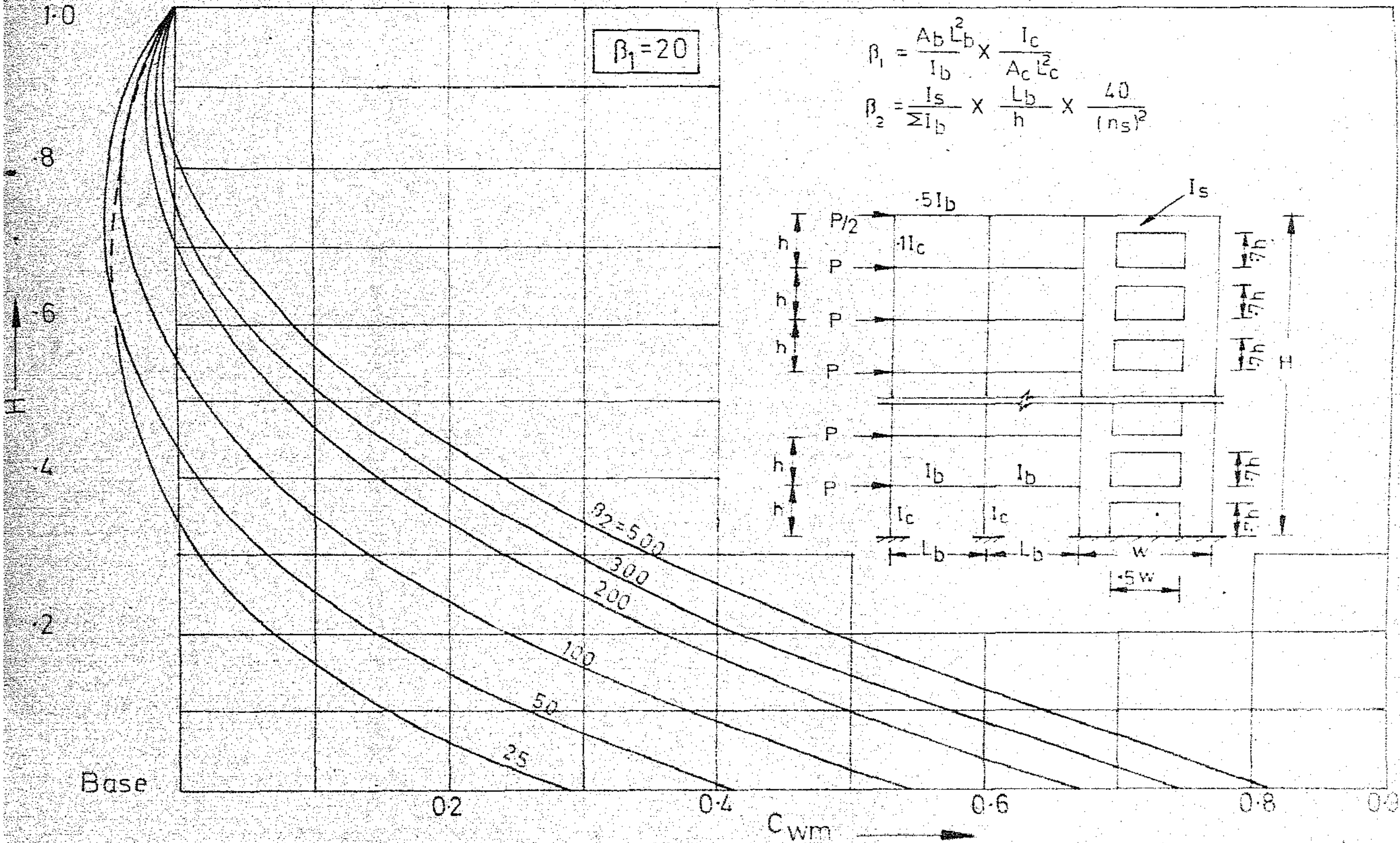


FIG. 5·20 a COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

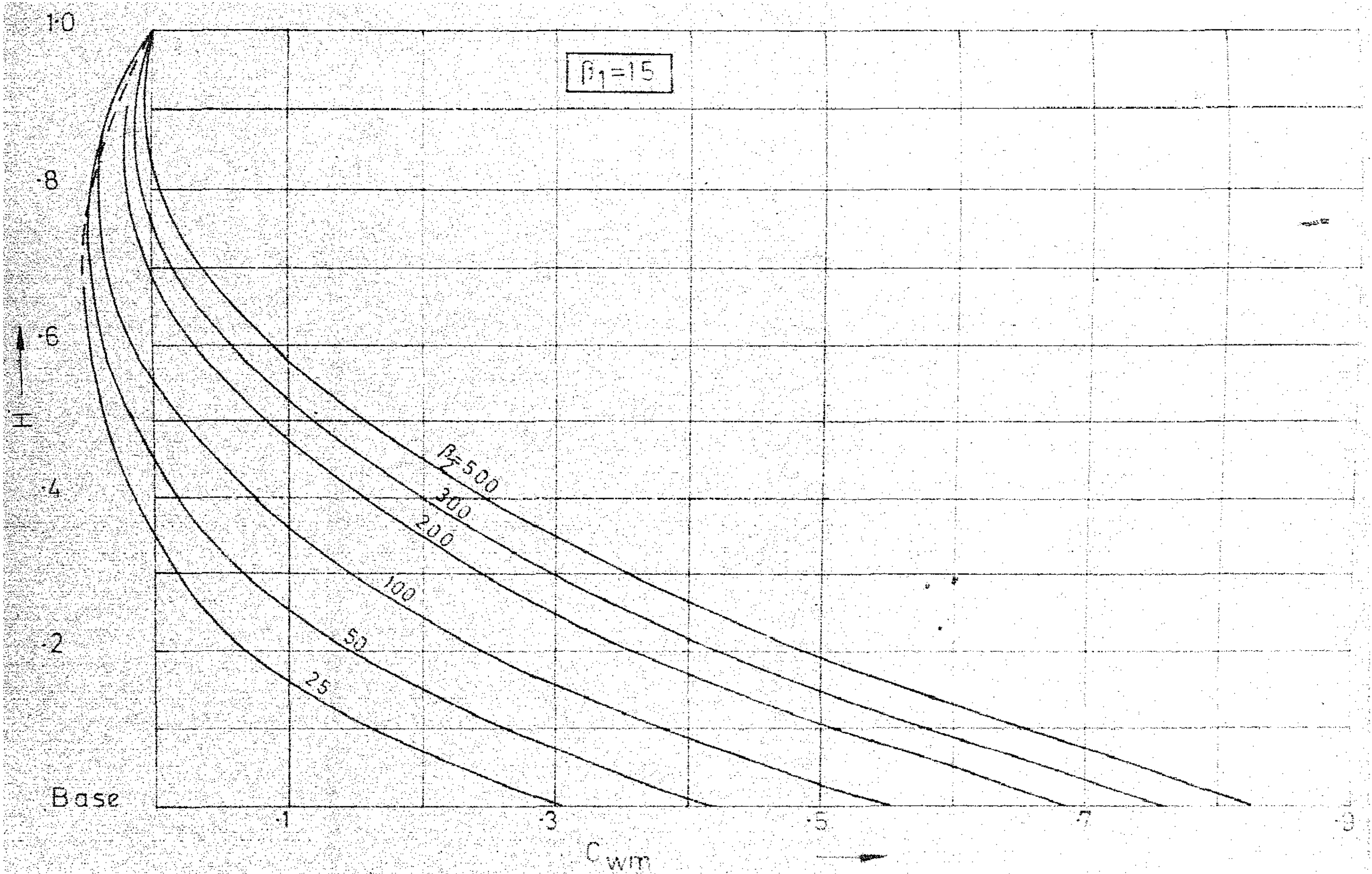


FIG. 5·20 b COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

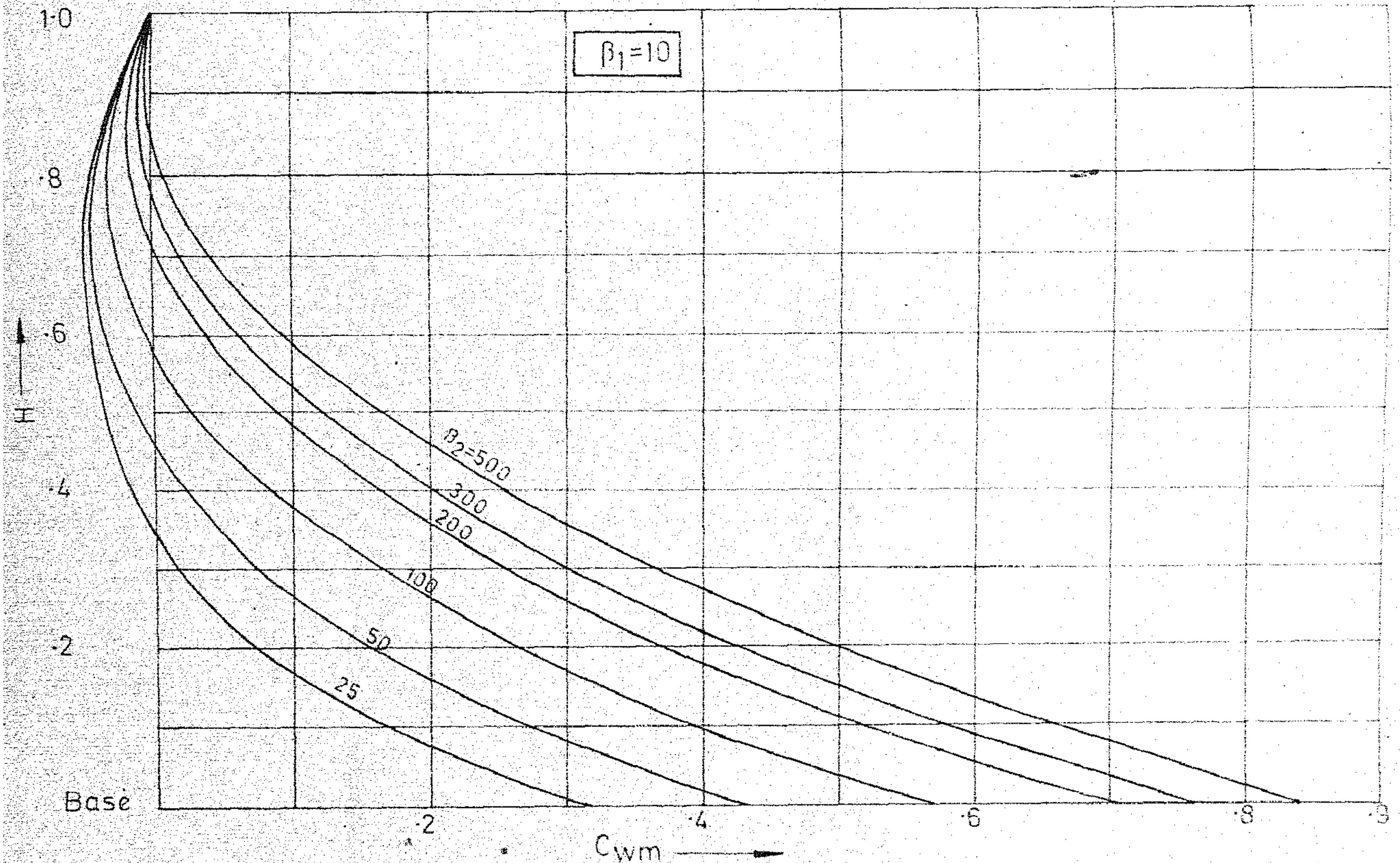


FIG. 5·20c COEFFICIENTS OF MOMENTS IN THE SHEAR WALL

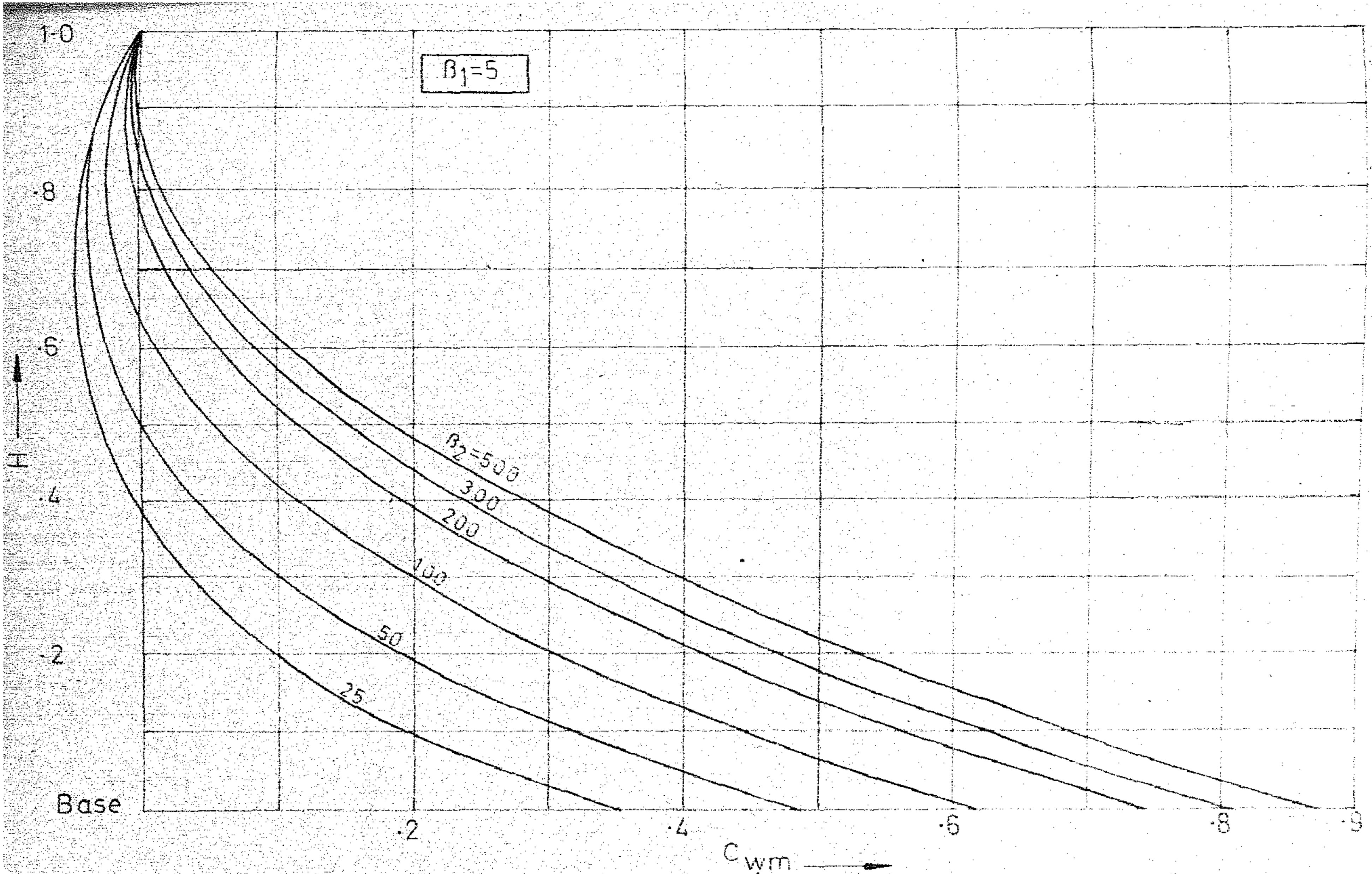


FIG. 5·20 d COEFFICIENTS OF MOMENT IN THE SHEAR WALL

5.8.3 DESIGN EXAMPLE

The use of the design curves is illustrated through an example. A two bay 15 storey frame with an interconnected shear wall (Fig. 5.21) in which, the stiffness factor of the frame, $\beta_1 = 10$ and the relative stiffness of the shear wall, $\beta_2 = 200$, is considered. The coefficients 'C_s' of the shear forces in different storeys of the frame are taken from the graphs for $\beta_1 = 10$ and $\beta_2 = 200$ (Fig. 5.14C) corresponding to the level of the mid-height of the storeys. For example, the mid-height of the fourth storey of the 15 storey frame is at $\frac{7}{30}$ H = 0.233 H, from base. The value of the coefficient 'C_s' for shear in the frame at 0.233 H, gives the shear force (V_{f4}) in the fourth storey as a fraction of the total applied shear V_{t4} = 11.5P, in this storey. The coefficients 'C_{im}' of the interaction moments on the frame are taken from the curves at levels corresponding to the height of the floor levels from the base (Fig.5.15C). Similarly, the coefficients C_{wm} of the moments on the shear wall at different levels are obtained from the graphs (Fig. 5.16C). The coefficients for shear in the frame, interaction moments on the frame and the moments on the walls of the 15 storey frame-wall system were also computed

$$\beta_1 = \frac{A_b L_b^2}{I_b} \times \frac{I_c}{A_c L_c^2} = 10 \quad ; \quad \beta_2 = \frac{I_s}{2\,\Sigma I_c} \times \frac{L_b}{h} \times \frac{40}{(ns)^2} = 200$$

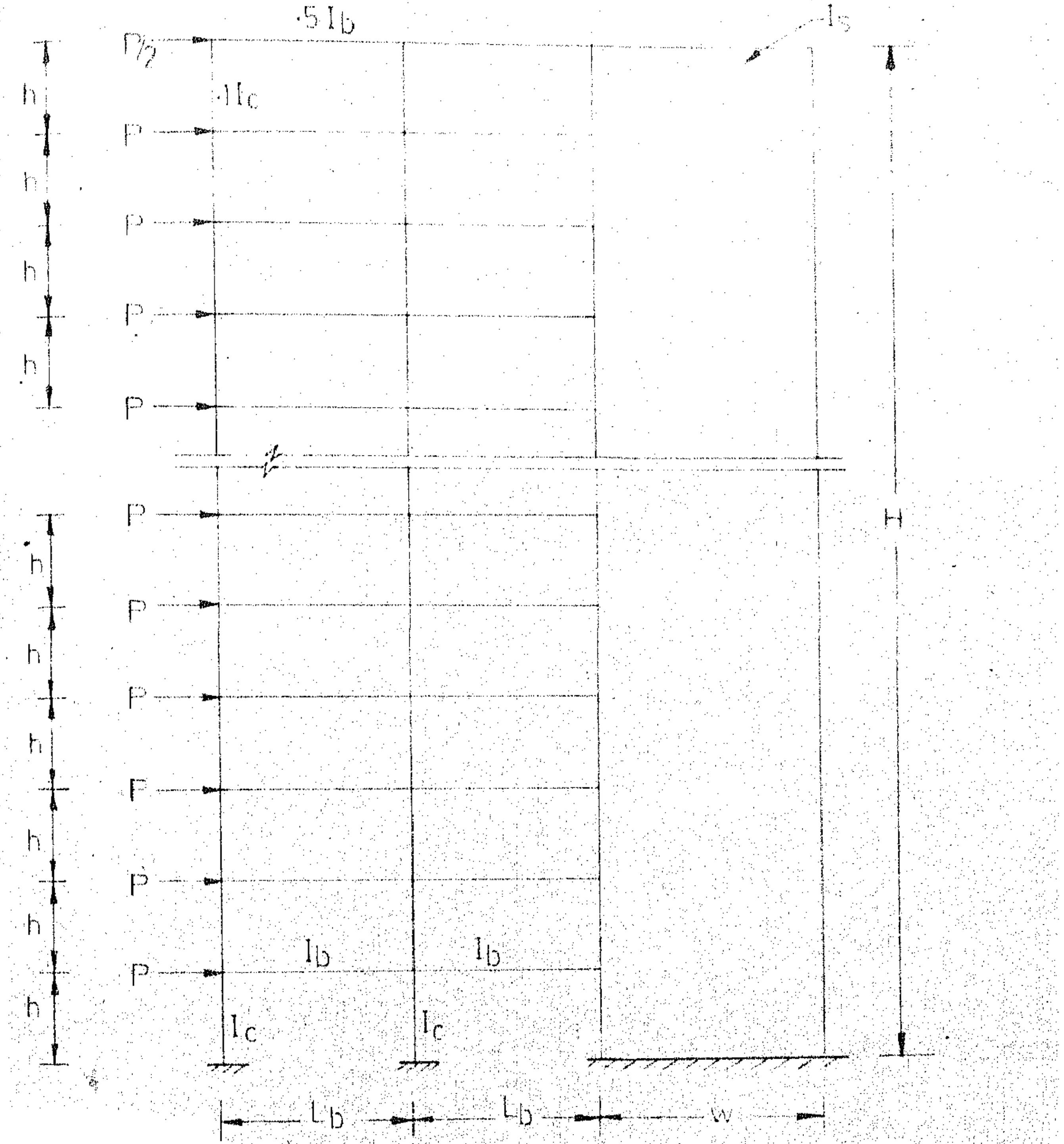


FIG. 5.21 15 STOREY FRAME AND INTERCONNECTED SHEAR WALL-EXAMPLE

by carrying out a separate interaction analysis. These coefficients obtained from the graph and the interaction analysis are compared in Tables 5.4 and 5.5. It can be noticed that the values of coefficients from the graphs match closely with the computed values.

With shear forces in each storey of the frame evaluated, the shear forces in the wall are obtained as the difference between the total applied shear and the frame shear in the storey. The lateral interaction forces on the frame at any floor level are obtained as the difference between the shear forces in the storeys just above and below that floor level. With the interaction forces viz. the lateral forces and moments on the frame known, the forces in the individual members of the frame can be evaluated by any convenient method of frame analysis. As the moments at different levels and the shear forces in different storeys of the wall are known, the design of the wall can be carried out in the usual way.

TABLE 5.4

SHEAR FORCES IN FRAME OF 15-STOREY

SHEAR WALL-FRAME SYSTEM (DESIGN EXAMPLE)

Storeys	Height from base to mid-storey level	Coefficients of shear forces in the frame (C_s)	
		Computed	From Graph
1	0.033 H	0.038	0.030
2	0.100 H	0.063	0.060
3	0.167 H	0.096	0.092
4	0.233 H	0.125	0.120
5	0.300 H	0.151	0.148
6	0.367 H	0.175	0.170
7	0.433 H	0.198	0.192
8	0.500 H	0.222	0.214
9	0.567 H	0.248	0.244
10	0.633 H	0.278	0.270
11	0.700 H	0.318	0.312
12	0.767 H	0.375	0.375
13	0.833 H	0.471	0.470
14	0.900 H	0.676	0.690
15	0.967 H	1.832	2.100

TABLE 5.5

INTERACTION MOMENTS ON FRAME AND MOMENTS ON WALL OF 15 STOREY SHEAR WALL-FRAME SYSTEM (DESIGN EXAMPLE)

Floor Levels	Coefficients of interaction moments on frame (C_{im})		Coefficients of moments on the wall (C_{wm})	
	Computed	From Graph	Computed	From Graph
Base	-	-	0.718	0.720
2	0.00205	0.00200	0.594	0.602
3	0.00355	0.00350	0.485	0.497
4	0.00466	0.00467	0.389	0.390
5	0.00542	0.00540	0.307	0.300
6	0.00590	0.00587	0.235	0.235
7	0.00615	0.00613	0.174	0.176
8	0.00622	0.00620	0.122	0.123
9	0.00615	0.00620	0.080	0.080
10	0.00596	0.00600	0.045	0.045
11	0.00569	0.00567	0.019	0.017
12	0.00536	0.00532	-0.0004	-0.0010
13	0.00499	0.00500	-0.012	-0.014
14	0.00459	0.00460	-0.017	-0.016
15	0.00416	0.00413	-0.014	-0.013
16	0.00337	0.00346	-0.005	-0.003

limiting sizes of the openings could be analysed by beam theory for the computation of displacements. The applicability of beam theory to the walls with openings is restricted not only by the overall sizes of the openings but also by their width. It is observed that walls with centrally placed openings whose area does not exceed 35 percent of the wall and also the width does not exceed half that of the wall, could be analysed using beam theory for the evaluation of the displacements. The comparison of the displacements of the solid shear walls, evaluated by the use of the finite element method and the beam theory, indicates that the rectangular element with two degrees of freedom per node used in the present work, is well suited for the analysis of shear wall structures.

6.3 NONDIMENSIONALIZED FRAME STIFFNESS METHOD

The non-dimensionalised form of the stiffness method used for the evaluation of the forces of interaction on the frame, offers convenience in the preparation of design curves for the interaction force coefficients of the shear wall-frame system. As only the relative dimensions of the members of the frame are involved, this procedure allows the designer to utilise the same results, for the analysis of frames with different absolute values

of beam and column dimensions but their relative values being same. The regular structural configuration of the frame facilitates the development of computer program for the interaction analysis, requiring core memory storages, mainly as a function of the number of bays, rather than the number of storeys.

6.4 NATURE OF THE INTERACTION FORCES

The structural components of the frame are basically flexural elements while the shear wall panel has very high shear stiffness. The interaction phenomenon of these two components having divergent structural behaviours makes the problem more interesting. The general distribution of the interaction forces between the shear wall and frame viz. moments, lateral and vertical forces, are such that the applied lateral loads are resisted by the frame and wall in an efficient manner. However, the lateral interaction forces are distributed in such manner that the frame pulls the shear wall and reduces the applied shear forces on the wall in the upper storeys while in the lower storeys the frame adds to the shear forces in the wall. Though the net effect is that the frame takes a part of the total lateral load, its behaviour is sharing and self balancing type. The interaction moments

and vertical forces counteract the applied loads uniformly along the height of the shear wall.

6.5 INFLUENCE OF DIFFERENT PARAMETERS

6.5.1 Axial Deformations of the Shear Wall

The approximations involved in neglecting the effect of axial deformations of the wall on the moments at the ends of the girders connected to the wall have been studied. It is observed that in most cases, this does not introduce appreciable errors. However, when the shear wall connected to the frame is relatively stiff, the interaction moments on the frame are under-estimated by neglecting the axial deformations of the shear wall.

6.5.2 Variation of Moment of Inertia of the Members of the Frame and Shear Wall

The lateral interaction forces between the frame and the shear wall are quite sensitive to the variations in the second moment of area of the column and beam members of the frame in different storeys along the height. This phenomenon indicates that caution should be excercised in the case of design curves prepared for a specified type of variation of the moments

of inertia of the members of the frame, while using them for a frame wall system with a very different type of variation of the moments of inertia.

Moderate reductions in the moment of inertia of the shear wall along the height do not affect the interaction behaviour of the shear wall frame system in any significant manner.

Studies were carried out to obtain an idea about the limiting ratio of stiffness of the shear wall to the frame, beyond which any increase in the stiffness of the wall does not effect the load carried by the wall. For a 20 storey frame-wall system (Fig. 5.2) analysed, the limiting ratio of the moment of inertia of the shear wall to the column is about 1130 (when the ratio of the flexural stiffness of column to beam in the frame is 5.3). Any further increase in the stiffness of the shear wall will not change the interaction forces to a significant value.

6.6 CONVERGENCE

The accuracy of the results obtained in the interaction analysis employing the iterative scheme can be improved by imposing strict convergence criteria. Out of the two extrapolation techniques viz. 'Forced conver-

gence technique' (29) and 'Aitken's δ^2 method' (61) tried, the Forced convergence technique offers assured convergence of the iterative scheme to the correct solution.

6.7 DESIGN CURVES

The design curves presented herein give the interaction forces on the frame and shear wall with or without openings, in terms of dimensionless coefficients for a wide range of structural proportions. When the relative stiffness ratios β_1 and β_2 of the shear wall and frame system fall in the ranges indicated in these charts, the interactions forces on the frame and the wall could be obtained without carrying out a detailed interaction analysis. Simple programmes for the analysis of frames are easily available. After determining the interaction forces on the frame of a given frame-wall system from the design curves, it is rather easy to analyse the frame with the help of any one of the available computer programs. The same thing can be achieved by writing a program using a more generalised displacement method. The example of the previous chapter illustrates the simplicity in accomplishing the analysis of a frame-wall system with the help of the design curves.

In many cases, a designer has to make a decision in advance about the relative sizes of the wall and frame. As the height of the building increases, the decision process becomes difficult. The design curves give an insight to the designer about the interaction behaviour of the frame-wall system for different relative stiffnesses of frame and wall. The designer has an advanced information from the charts presented herein which enable him to come up with a rational decision without many trials.

6.8 SUGGESTIONS FOR FURTHER WORK

It appears that there are no design curves available to get the interaction forces between the frame and interconnected coupled shear wall. Preparation of charts giving the interaction forces for different relative stiffnesses of frame and coupled shear wall structures will be a welcome addition to the family of design curves available for shear wall-frame structures.

The research work on the dynamic analysis of shear wall and frame structures is limited. When the shear walls have openings, it may be expected that much higher stresses would be induced around the openings under dynamic loads as compared to the case of static loads.

Extensive experimental as well as theoretical studies will have to be undertaken to get a better understanding of the behaviour of the combined wall and frame systems under dynamic loads.

The role played by the floor diaphragms in sharing a part of the lateral loads applied on the shear wall-frame system has not been assessed well. The amount of work available regarding the influence of the inplane deformations of the floor diaphragms on the interaction behaviour is limited. There is ample scope to carry out further studies on this aspect of slab-wall interaction. The analysis for internal forces in the shear walls combined with floor diaphragms where the walls also act as load bearing walls, requires further investigations.

Many investigators have undertaken research on the non-linear behaviour and ultimate strength aspects of shear wall structures but the studies are in the initial stage. It can be hoped that these studies would help in the rational evaluation of load factors for the design of shear wall structures in the near future.

Development of procedures for an optimal design of shear wall and interconnected frame systems, in terms of the relative sizes of the component wall

6.9 ADDITIONAL COMMENTS

6.9.1 Validity of Beam Theory

The thesis presents two sets of interaction analyses. In one analysis the shear wall with or without openings is assumed to be a beam and the stiffness matrix is generated by using beam theory including shear distorsion. Whereas in the second set a finite element analysis is used for the same shear wall. The purpose of developing the parallel programmes is two-fold :

1. To find out the limits of validity of the beam theory in the interaction problem with respect to the size of openings.
2. The finite element analysis of the interaction programme required a large computer time. Since the main object of the problem is to generate design coefficients for a large set of non-dimensionalized parameters, the computer time required for such a purpose would be enormous. Considerable computer time can be saved by using simple beam theory programme wherever the accuracy of its solution is within limits.

Several examples have been solved by using the two programmes. Deformations as well as interaction forces obtained from these different programmes were compared. In first set of examples, only deformation analysis was done by

both the methods, taking the opening of shear wall as a variable parameter. In this set of examples, three loads were applied at each joint location and the corresponding deformations at each of the joints were compared. Some comparisons of typical graphs were presented in the previous sections. The results from beam theory checked very closely with those of the finite element analysis, if the area of opening is 35% of the shear wall and the width of the opening is less than 50% of that of the wall. This illustrates that for an arbitrary set of loads, the deformations check but does not guarantee that this behaviour will continue even for any typical interaction forces. To check for such a situation, ten and twenty storey shear wall-frame systems were analysed for interaction forces and deformations by using finite element method. The deformations at the interconnected points of the shear wall and frame were substituted in the stiffness equation of the simple beam theory. The joint forces thus obtained from the beam theory due to the deformations corresponding to the finite element programme were compared with interaction forces of the finite element solutions. These solutions compared well within the limits of the openings of shear walls already mentioned.

Many examples have been tested and found to give satisfactory results, therefore it is more rational to use a beam theory upto the limits for which it is applicable. The

finite element programme must be used for dependable solutions, if the area of the opening is more than 35% and, the width of the opening is more than 50% of the wall width. Areas of door and window openings of most shear walls will be less than 35% of the shear wall. Therefore the interaction force coefficients are generated by using simple beam theory instead of the finite element method. This has saved several hours of computer time.

6.9.2 Validity and Limitation of the Design Curve

Only two sets of interaction force coefficient curves are presented. In each set there are several non-dimensionalized variables associated with relative dimensions of two bay frame and shear wall. One set of curves gives the interaction force coefficients for solid shear wall while the other set gives for a shear wall with opening of 35% . A 35% opening in the middle line of a shear wall with width of opening not exceeding 50% of the wall width, reduces the bending stiffness coefficients of a panel to a maximum extent of 14% . Such a reduction in the elements of stiffness matrix of the shear wall with opening has a tendency to increase the lateral loading on the frame. The lateral interaction load on the frame is negative at lower floor levels while it is positive at the upper floor levels. This type of partial self-balancing tendency of the interaction forces is not much

affected by the marginal variation in the stiffness matrix. Difference in the interaction force coefficients of a solid shear wall and the shear wall with openings upto 35% area is in the order of about 10% . The interaction force coefficients for 20% and 30% openings were also computed for typical frame systems and found to lie in between the coefficients of the solid wall and 35% opening shear wall. Even though the curves given are for solid shear wall and shear wall with 35% openings, a designer can obtain these coefficients for any other arbitrary openings of a shear wall by simple linear interpolation. The curves presented can be used for a very wide range of shear wall-frame systems, having any arbitrary openings and arbitrary number of floors. However there are certain limits beyond which the curves are not helpful. The limits are : (a) the area of the opening in shear wall should not exceed 35% of the wall area, (b) the width of the opening should not exceed 50% of the width of the wall, and (c) the number of bays in a frame has to be only two. The first two limits are not really limits since most openings of doors and windows tend to be within these limits. The number of bays is a real limitation. The computer programme developed is capable of accommodating any number of bays. Presentation of design curves taking number of bays as a parameter, multiplies the number of pages of the thesis. It has been found that frames having more than two bays tend to be very wide.

Such frames do not require shear walls. Shear wall and one bay frame systems are also not common in practice. Such slender buildings are often designed with staggered shear walls and frames. Slender buildings are often provided with wide shear walls at the ends of the building with intermediate frames. Shear wall-frame systems are normally used in three bays width buildings in which one of the bays is provided with shear wall while the other two bays are provided with frame. Hence only shear wall and two bay frame systems are considered in the present.work.

The interaction force coefficients can be applied to staggered frame and shear wall frame systems without any loss of accuracy. If a building has shear walls with frames at a limited number of intervals while at other intervals only frames are provided, the thicknesses of all the frames and the shear walls could be added respectively and then this modified shear wall frame system could be analysed. Floor slab is rigid in plane of the floor, so each bay no matter whether a shear wall is provided or not in that bay, will deform uniformly. If the shear walls are placed symmetrically along the length of a building and the building is not subjected to torsion, then these interaction force coefficients are applicable without loss of any accuracy in the solution. In case of buildings subjected to torsion, these coefficients are still good with torsional correction applied

to the three dimensional frame system.

6.9.3 Interaction Force Distribution

An interaction force distribution pattern for various proportions of column and shear wall systems with any uniform variation in the moment of inertia of the columns is found to be consistent. A typical interaction force distribution with a uniform variation in the moment of inertia of columns is shown in Fig. 5.7. This distribution pattern is very similar to those already reported in literature. There is a positive as well as negative interaction load on the frame. The negative interaction lateral force which occurs in the lower floors is the resisting force while the positive interaction force in the upper floors is an additive load on the frame. The total interaction lateral force on frame may be divided into two parts: (a) Effective lateral force, (which is equal to the sum of the frame interaction forces) is positive. This indicates the frame shares the external load acting on the system. (b) The self balancing type of lateral force distribution generates a moment which compensates the moment caused by external load. (The positive interaction lateral force in the upper floors when combined with an equal and opposite interaction force in the lower floors produces a compensating bending moment). This type of action has been reported in case of coupled shear walls also. However Fig. 5.8 indicates a deviation of the distribution pattern of the interaction force

when the moment of inertia of the columns is abruptly changed at floor levels. An average line drawn through this oscillatory type of behaviour of the interaction force, tends to that of Fig. 5.7. Several examples with moment of inertia varying as a step function have indicated a similar oscillatory type behaviour. However behaviour pattern of interaction forces using a modified uniform variation in the moment of inertia of columns by taking mean values of the moment of inertia of the columns at floor levels indicated a normal behaviour. The nondimensionalized computer programme of frame analysis has been tested for its accuracy with the help of a standard programme based on Kani's method. The results checked well. The frame analysis has been found to be free of errors. However a sudden jump in the moment of inertia of columns at any floor level introduced sharp reversal in the interaction forces of the shear wall-frame interaction programme. This may be a move of numerical error in the programme than an actual behaviour. The design interaction force coefficients presented are therefore based on uniform variation in the moment of inertia of the columns.

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Design interaction force coefficients of shear wall-frame systems.

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